equivalence relation*

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An *equivalence relation* $\sim$ on a set S is a relation that is:

Reflexive. $a \sim a$ for all $a \in S$.

Symmetric. Whenever $a \sim b$, then $b \sim a$.

Transitive. If $a \sim b$ and $b \sim c$ then $a \sim c$.

If a and b are related this way we say that they are *equivalent* under $\sim$. If $a \in S$, then the set of all elements of S that are equivalent to a is called the *equivalence class* of a. The set of all equivalence classes under $\sim$ is written $S/\sim$.

An equivalence relation on a set induces a partition on it. Conversely, any partition induces an equivalence relation. Equivalence relations are important, because often the set S can be 'transformed' into another set (quotient space) by considering each equivalence class as a single unit.

Two examples of equivalence relations:

1. Consider the set of integers $\mathbb{Z}$ and take a positive integer m. Then m induces an equivalence relation by $a \sim b$ when m divides $b - a$ (that is, a and b leave the same remainder when divided by m).

2. Take a group $(G, \cdot)$ and a subgroup H. Define $a \sim b$ whenever $ab^{-1} \in H$. That defines an equivalence relation. Here equivalence classes are called cosets.

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