

# proof of the fundamental theorem of algebra (Liouville's theorem)\*

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Let  $f: \mathbb{C} \rightarrow \mathbb{C}$  be a polynomial, and suppose  $f$  has no root in  $\mathbb{C}$ . We will show  $f$  is constant.

Let  $g = \frac{1}{f}$ . Since  $f$  is never zero,  $g$  is defined and holomorphic on  $\mathbb{C}$  (ie. it is entire). Moreover, since  $f$  is a polynomial,  $|f(z)| \rightarrow \infty$  as  $|z| \rightarrow \infty$ , and so  $|g(z)| \rightarrow 0$  as  $|z| \rightarrow \infty$ . Then there is some  $M > 0$  such that  $|g(z)| < 1$  whenever  $|z| > M$ , and  $g$  is continuous and so bounded on the compact set  $\{z \in \mathbb{C} : |z| \leq M\}$ .

So  $g$  is bounded and entire, and therefore by Liouville's theorem  $g$  is constant. So  $f$  is constant as required.  $\square$

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