Chebyshev functions*

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There are two different functions which are collectively known as the *Chebyshev functions*:

$$\vartheta(x) = \sum_{p \le x} \log p.$$

where the notation used indicates the summation over all positive primes p less than or equal to x, and

$$\psi(x) = \sum_{p \le x} k \log p,$$

where the same summation notation is used and k denotes the unique integer such that $p^k \le x$ but $p^{k+1} > x$. Heuristically, the first of these two functions measures the number of primes less than x and the second does the same, but weighting each prime in accordance with their logarithmic relationship to x.

Many innocuous results in number theory owe their proof to a relatively simple analysis of the asymptotics of one or both of these functions. For example, the fact that for any n, we have

$$\prod_{p \le n} p < 4^n$$

is equivalent to the statement that $\vartheta(x) < x \log 4$.

A somewhat less innocuous result is that the prime number theorem (i.e., that $\pi(x) \sim \frac{x}{\log x}$) is equivalent to the statement that $\vartheta(x) \sim x$, which in turn, is equivalent to the statement that $\psi(x) \sim x$.

References

[1] Ireland, Kenneth and Rosen, Michael. A Classical Introduction to Modern Number Theory. Springer, 1998.

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[2] Nathanson, Melvyn B. Elementary Methods in Number Theory. Springer, 2000.