A positive integer \( m \) is called a \textit{practical number} if every positive integer \( n < m \) is a sum of distinct positive divisors of \( m \).

\textbf{Lemma.} An integer \( m \geq 2 \), \( m = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_\ell^{\alpha_\ell} \), with primes \( p_1 < p_2 < \cdots < p_\ell \) and integers \( \alpha_i \geq 1 \), is practical if and only if \( p_1 = 2 \) and, for \( i = 2, 3, \ldots, \ell \),

\[ p_i \leq \sigma(p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_i^{\alpha_i-1}) + 1, \]

where \( \sigma(n) \) denotes the sum of the positive divisors of \( n \).

Let \( P(x) \) be the counting function of practical numbers. Saias [?], using suitable sieve methods introduced by Tenenbaum [?, ?], proved a good estimate in terms of a Chebyshev-type theorem: for suitable constants \( c_1 \) and \( c_2 \),

\[ c_1 \frac{x}{\log x} < P(x) < c_2 \frac{x}{\log x}. \]

In [?] Melfi proved a Goldbach-type result showing that every even positive integer is a sum of two practical numbers, and that there exist infinitely many triplets of practical numbers of the form \( m - 2, m, m + 2 \).

\textbf{References}


