area functions*

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The most usual *area functions*:

- The inverse function of the hyperbolic sine (in Latin *sinus hyperbolicus*) is arsinh (*area sini hyperbolici*):

$$\operatorname{arsinh} x := \ln\left(x+\sqrt{x^2+1}\right)$$

- The inverse function of the hyperbolic cosine (in Latin *cosinus hyperbolicus*) is arcosh (*area cosini hyperbolici*):

$$\operatorname{arcosh} x := \ln(x+\sqrt{x^2-1})$$

  It is defined for $x \geqq 1$.

- The inverse function of the hyperbolic tangent (in Latin *tangens hyperbolica*) is artanh (*area tangentis hyperbolicae*):

$$\operatorname{artanh} x := \frac{1}{2}\ln\frac{1+x}{1-x}$$

  It is defined for $-1 < x < 1$.

- The inverse function of the hyperbolic cotangent (in Latin *cotangens hyperbolica*) is arcoth (*area cotangentis hyperbolicae*):

$$\operatorname{arcoth} x := \frac{1}{2}\ln\frac{x+1}{x-1}$$

  It is defined for $|x| > 1$.

These four functions are denoted also by $\sinh^{-1} x$, $\cosh^{-1} x$, $\tanh^{-1} x$ and $\coth^{-1} x$.

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Derivatives:

$$\frac{d}{dx}\operatorname{arsinh} x = \frac{1}{\sqrt{x^2+1}}$$
$$\frac{d}{dx}\operatorname{arcosh} x = \frac{1}{\sqrt{x^2-1}}$$
$$\frac{d}{dx}\operatorname{artanh} x = \frac{1}{1-x^2}$$
$$\frac{d}{dx}\operatorname{arcoth} x = \frac{1}{1-x^2}$$

The functions arsinh and artanh have the simple Taylor series

$$\operatorname{arsinh} x = x - \frac{1}{2}\cdot\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\cdot\frac{x^5}{5} - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\cdot\frac{x^7}{7} + - \cdots \quad (|x| \leqq 1),$$

$$\operatorname{artanh} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \cdots \quad (|x| < 1).$$

Because the inverse tangent function (see the cyclometric functions) has the expansion $\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + - \cdots$ $(|x| \leqq 1)$, we see that

$$\operatorname{artanh} x = \frac{1}{i}\arctan ix;$$

similarly we get

$$\operatorname{arsinh} x = \frac{1}{i}\arcsin ix.$$

Some other formulae which may be obtained by means of the addition formulae of the hyperbolic functions:

$$\operatorname{arsinh} x \pm \operatorname{arsinh} y = \operatorname{arsinh}(x\sqrt{y^2+1} \pm y\sqrt{x^2+1})$$
$$\operatorname{arcosh} x \pm \operatorname{arcosh} y = \operatorname{arcosh}(xy \pm \sqrt{x^2-1}\sqrt{y^2-1})$$
$$\operatorname{artanh} x \pm \operatorname{artanh} y = \operatorname{artanh}\frac{x \pm y}{1 \pm xy}$$

The classic abbreviations "arsinh" and "arcosh" are explained as follows: The unit hyperbola $x^2-y^2 = 1$ (its right half) has the parametric representation

$$\begin{cases} x = \cosh A, \\ y = \sinh A; \end{cases}$$

here A means the area bounded by the hyperbola and the straight line segments OP and OQ, where O is the origin, P is the point $(x,\, y)$ of the hyperbola and Q is the point $(x,\, -y)$ of the hyperbola. Thus, conversely, A is the area having hyperbolic cosine equal to x (*area cosini hyperbolici* x), similarly A is the area having hyperbolic sine equal to y (*area sini hyperbolici* y).

Note. In some countries the abbreviation "ar" in the symbols arsinh etc. is replaced by "a", "Ar", "arc" or "arg".