

category isomorphism*

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Let \mathcal{C} and \mathcal{D} be categories. An *isomorphism* $T : \mathcal{C} \rightarrow \mathcal{D}$ is a (covariant) functor which has a *two-sided inverse*. In other words, there is a (covariant) functor $S : \mathcal{D} \rightarrow \mathcal{C}$ such that $T \circ S = I_{\mathcal{D}}$ and $S \circ T = I_{\mathcal{C}}$, where $I_{\mathcal{D}}$ and $I_{\mathcal{C}}$ are the identity functors of \mathcal{D} and \mathcal{C} respectively. Two categories \mathcal{C} and \mathcal{D} are *isomorphic* if there exists a functor $T : \mathcal{C} \rightarrow \mathcal{D}$ that is an isomorphism.

Remarks

1. An isomorphism (functor) from \mathcal{C} to \mathcal{D} is just an isomorphism (in the sense of morphism) in the functor category $\mathcal{D}^{\mathcal{C}}$.
2. Two isomorphic categories are equivalent. The converse is not true. For example, the category of all finite sets is equivalent to its subcategory of all finite ordinals. But clearly these two categories are not isomorphic. Isomorphism has a “size” restriction, whereas natural equivalence does not.

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