Kronecker symbol*

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The **Kronecker symbol** is a generalization of the Jacobi symbol to all integers.

Let n be an integer, with prime factorization $u \cdot {p_1}^{e_1} \cdots {p_k}^{e_k}$, where u is a unit and the p_i are primes. Let $a \geq 0$ be an integer. The Kronecker symbol $\left(\frac{a}{n}\right)$ is defined to be

$$\left(\frac{a}{n}\right) = \left(\frac{a}{u}\right) \prod_{i=1}^{k} \left(\frac{a}{p_i}\right)^{e_i}$$

For odd p_i, the number $\left(\frac{a}{p_i}\right)$ is simply the usual Legendre symbol. This leaves the case when $p_i = 2$. We define $\left(\frac{a}{2}\right)$ by

$$\left(\frac{a}{2}\right) = \begin{cases} 0 & \text{if } a \text{ is even} \\ 1 & \text{if } a \text{ is odd and } n \equiv 1 \text{ or } n \equiv 7 \pmod 8 \\ -1 & \text{if } a \text{ is odd and } n \equiv 3 \text{ or } n \equiv 5 \pmod 8 \end{cases}$$

Since it extends the Jacobi symbol, the quantity $\left(\frac{a}{u}\right)$ is simply 1 when $u = 1$. When $u = -1$, we define it by

$$\left(\frac{a}{-1}\right) = \begin{cases} -1 & \text{if } a < 0 \\ 1 & \text{if } a > 0 \end{cases}$$

These extensions suffice to define the Kronecker symbol for all integer values n.

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