

# valuation determined by valuation domain\*

*pahio*<sup>†</sup>

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**Theorem.** Every valuation domain determines a Krull valuation of the field of fractions.

*Proof.* Let  $R$  be a valuation domain,  $K$  its field of fractions and  $E$  the group of units of  $R$ . Then  $E$  is a normal subgroup of the multiplicative group  $K^* = K \setminus \{0\}$ . So we can form the factor group  $K^*/E$ , consisting of all cosets  $aE$  where  $a \in K^*$ , and attach to it the additional “coset”  $0E$  getting thus a multiplicative group  $K/E$  equipped with zero. If  $\mathfrak{m} = R \setminus E$  is the maximal ideal of  $R$  (any valuation domain has a unique maximal ideal — cf. valuation domain is local), then we denote  $\mathfrak{m}^* = \mathfrak{m} \setminus \{0\}$  and  $S = \mathfrak{m}^*/E = \{aE : a \in \mathfrak{m}^*\}$ . Then the subsemigroup  $S$  of  $K/E$  makes  $K/E$  an ordered group equipped with zero. It is not hard to check that the mapping

$$x \mapsto |x| := xE$$

from  $K$  to  $K/E$  is a Krull valuation of the field  $K$ .

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