valuation determined by valuation domain*

pahio[†]

2013-03-21 18:29:14

Theorem. Every valuation domain determines a Krull valuation of the field of fractions.

Proof. Let R be a valuation domain, K its field of fractions and E the group of units of R. Then E is a normal subgroup of the multiplicative group $K^* = K \smallsetminus \{0\}$. So we can form the factor group K^*/E, consisting of all cosets aE where $a \in K^*$, and attach to it the additional "coset" $0E$ getting thus a multiplicative group K/E equipped with zero. If $\mathfrak{m} = R \smallsetminus E$ is the maximal ideal of R (any valuation domain has a unique maximal ideal — cf. valuation domain is local), then we denote $\mathfrak{m}^* = \mathfrak{m} \smallsetminus \{0\}$ and $S = \mathfrak{m}^*/E = \{aE : a \in \mathfrak{m}^*\}$. Then the subsemigroup S of K/E makes K/E an ordered group equipped with zero. It is not hard to check that the mapping

$$x \mapsto |x| := xE$$

from K to K/E is a Krull valuation of the field K.

*⟨*ValuationDeterminedByValuationDomain*⟩ created: ⟨*2013-03-21*⟩ by: ⟨*pahio*⟩ version: ⟨*36602*⟩ Privacy setting: ⟨*1*⟩ ⟨*Theorem*⟩ ⟨*13F30*⟩ ⟨*13A18*⟩ ⟨*12J20*⟩ ⟨*11R99*⟩