submodule*

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2013-03-21 19:10:40

Given a ring R and a left R-module T, a subset A of T is called a (*left*) *submodule* of T, if $(A, +)$ is a subgroup of $(M, +)$ and $ra \in A$ for all elements r of R and a of A.

Examples

1. The subsets $\{0\}$ and T are always submodules of the module T.
2. The set $\{t \in T : \ rt = t \ \forall r \in R\}$ of all invariant elements of T is a submodule of T.
3. If $X \subseteq T$ and $\mathfrak{a}$ is a left ideal of R, then the set

$$\mathfrak{a}X := \{\text{finite} \sum_{\nu} a_\nu x_\nu : \ a_\nu \in \mathfrak{a}, \ x_\nu \in X \ \forall \nu\}$$

is a submodule of T. Especially, RX is called the submodule *generated* by the subset X; then the elements of X are *generators* of this submodule.

There are some operations on submodules. Given the submodules A and B of T, the *sum* $A + B := \{a + b \in T : \ a \in A \wedge b \in B\}$ and the intersection $A \cap B$ are submodules of T.

The notion of sum may be extended for any family $\{A_j : \ j \in J\}$ of submodules: the sum $\sum_{j \in J} A_j$ of submodules consists of all finite sums $\sum_j a_j$ where every a_j belongs to one A_j of those submodules. The sum of submodules as well as the intersection $\bigcap_{j \in J} A_j$ are submodules of T. The submodule RX is the intersection of all submodules containing the subset X.

If T is a ring and R is a subring of T, then T is an R-module; then one can consider the *product* and the *quotient* of the left R-submodules A and B of T:

- $AB := \{\text{finite} \sum_\nu a_\nu b_\nu : \ a_\nu \in A, \ b_\nu \in B \ \forall \nu\}$
- $[A : B] := \{t \in T : \ tB \subseteq A\}$

Also these are left R-submodules of T.

*⟨*Submodule*⟩ created: ⟨*2013-03-21*⟩ by: ⟨*PrimeFan*⟩ version: ⟨*37040*⟩ Privacy setting: ⟨*1*⟩ ⟨*Definition*⟩ ⟨*20-00*⟩ ⟨*16-00*⟩ ⟨*13-00*⟩