Let $G$ be a non-abelian group with center $Z(G)$. Associate a graph $\Gamma_G$ with $G$ whose vertices are the non-central elements $G \setminus Z(G)$ and whose edges join those vertices $x, y \in G \setminus Z(G)$ for which $xy \neq yx$. Then $\Gamma_G$ is said to be the non-commuting graph of $G$. The non-commuting graph $\Gamma_G$ was first considered by Paul Erdős, when he posed the following problem in 1975 [B.H. Neumann, A problem of Paul Erdős on groups, J. Austral. Math. Soc. Ser. A 21 (1976), 467-472]: Let $G$ be a group whose non-commuting graph has no infinite complete subgraph. Is it true that there is a finite bound on the cardinalities of complete subgraphs of $\Gamma_G$?

B. H. Neumann answered positively Erdős’ question.

The non-commuting graph $\Gamma_G$ of a non-abelian group $G$ is always connected with diameter 2 and girth 3. It is also Hamiltonian. $\Gamma_G$ is planar if and only if $G$ is isomorphic to the symmetric group $S_3$, or the dihedral group $D_8$ of order 8 or the quaternion group $Q_8$ of order 8.


Examples
Symmetric group $S_3$

The symmetric group $S_3$ is the smallest non-abelian group. In cycle notation, we have $S_3 = \{(), (12), (13), (23), (123), (132)\}$.

The center is trivial: $Z(S_3) = \{()\}$. The non-commuting graph in Figure ?? contains all possible edges except one.
Figure 1: Non-commuting graph of the symmetric group $S_3$

Figure 2: Non-commuting graph of the octic group

**Octic group**

The dihedral group $D_8$, generally known as the octic group, is the symmetry group of the square. If you label the vertices of the square from 1 to 4 going along the edges, the octic group may be seen as a subgroup of the symmetric group $S_4$:

$$D_8 := \{((), (13), (24), (12)(34), (13)(24), (14)(23), (1234), (1432))\}.$$  

So the octic group has order 8 (hence its name), and its center consists of the identity element and the simultaneous flip around both diagonals $(13)(24)$. Its non-commuting graph is given in Figure ??.