

absorbing set*

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Let V be a vector space over a field F equipped with a non-discrete valuation $|\cdot| : F \rightarrow \mathbb{R}$. Let A, B be two subsets of V . Then A is said to *absorb* B if there is a non-negative real number r such that, for all $\lambda \in F$ with $|\lambda| \geq r$, $B \subseteq \lambda A$. A is said to be an *absorbing set*, or a *radial subset* of V if A absorbs all finite subsets of V .

Equivalently, A is absorbing if for any $x \in V$, there is a non-negative real number r such that $x \in \lambda A$ for all $\lambda \in F$ with $|\lambda| \geq r$. If a finite subset B of V consists of x_1, \dots, x_n , then corresponding to each x_i , there is an $r_i \geq 0$ such that $x_i \in \lambda A$ such that $|\lambda| \geq r_i, \forall \lambda \in F$. So $x_i \in \lambda A$ with $|\lambda| \geq r$ if we take $r = \max\{r_1, \dots, r_n\}$. So A absorbs B .

Example. If $V = \mathbb{R}^n$ and $F = \mathbb{R}$, then any set containing an open ball centered at 0 is absorbing. This implies that an absorbing set does not have to be connected, convex.

A closely related concept is that of a *circled set*, or a *balanced set*. Let V and F be defined as above. A subset C of V is said to be *circled*, or *balanced*, if $\lambda C \subseteq C$ for all $|\lambda| \leq 1$. Clearly, C absorbs itself ($C \subseteq \lambda^{-1}C, |\lambda^{-1}| \geq 1$), and $0 \in C$. C is also symmetric ($-C = C$), for $-C \subseteq C$ and $C = -(-C) \subseteq -C$. As an example of a circled set that is neither absorbing nor convex, consider $V = \mathbb{R}^2$ and $F = \mathbb{R}$, and C the union of x and y axes. For an example of an absorbing set that is not circled, take the union of a unit disk and an annulus centered at 0 that is large enough so it is disjoint from the disk.

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