equation*

pahio[†]

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Equation

An *equation* concerns usually elements of a certain set M, where one can say if two elements are equal. In the simplest case, M has one binary operation "$*$" producing as result some elements of M, and these can be compared. Then, an equation in $(M,\,*)$ is a proposition of the form

$$E_1 = E_2, \tag{1}$$

where one has *equated* two expressions E_1 and E_2 formed with "$*$" of the elements or indeterminates of M. We call the expressions E_1 and E_2 respectively the *left hand side* and the *right hand side* of the equation (1).

Example. Let S be a set and 2^S the set of its subsets. In the groupoid $(2^S,\,\smallsetminus)$, where "$\smallsetminus$" is the set difference, we can write the equation

$$(A\smallsetminus B)\smallsetminus B = A\smallsetminus B$$

(which is always true).

Of course, M may be equipped with more operations or be a module with some ring of multipliers — then an equation (1) may contain them.

But one need not assume any algebraic structure for the set M where the expressions E_1 and E_2 are values or where they represent generic elements. Such a situation would occur e.g. if one has a continuous mapping f from a topological space L to another M; then one can consider an equation

$$f(x) = y.$$

A somewhat comparable case is the equation

$$\dim V = 2$$

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where V is a certain or a generic vector space; both sides represent elements of the extended real number system.

Root of equation

If an equation (1) in M contains one indeterminate, say x, then a value of x which satisfies (1), i.e. makes it true, is called a *root* or a *solution* of the equation. Especially, if we have a polynomial equation $f(x) = 0$, we may speak of the *multiplicity* or the *order of a root* x_0; it is the multiplicity of the zero x_0 of the polynomial $f(x)$. A *multiple root* has multiplicity greater than 1.

Example. The equation

$$x^2+1 = x$$

in the system $\mathbb{C}$ of the complex numbers has as its roots the numbers

$$x := \frac{1\pm i\sqrt{3}}{2},$$

which, by the way, are the primitive sixth roots of unity. Their multiplicities are 1.