variable*

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The word *variable* as used in mathematics (and in other scientific fields that use mathematics) is somewhat vague and may have different meanings depending on the context. Variables are usually denoted by a single Roman or Greek letter, e.g. x, although sometimes a whole word or phrase can be used also.

Here is a list of some of the meanings of *variable*:

(i) As "mathematical" variables. These stand for a concrete object, for example, an element of the real numbers That is, when we write the symbol x, it is a stand-in for various numbers: e.g. $2, 3, \pi, e, 578.24$. But we do not name these numbers specifically, because we may want to talk about all these numbers at once, in a general statement, theorem, or proof about numbers.

Sense (i) is probably the most common usage in mainstream mathematics.

(ii) As placeholders in functional notation. For example, we may be defining a function using the phrase "define the function $f(z) = z^2 + 4$ for complex numbers z. This usage of a variable is slightly different from sense (i), because our objective is to talk about the *function f, and not its value* at a number z which is $f(z)$. The notation "$f(z) = z^2 + 4$" is merely a much more convenient way of saying: "define the function f which takes a complex number, multiplies it by itself, and then adds four to it". It could also be rephrased this way: "define a function f such that the statement $f(z) = z^2 + 4$ is true for all complex numbers z (in sense (i))".

On the other hand, the symbol f, if we were to contemplate it as a "variable", arguably belongs to the sense (i); in this case we are talking about *some specific function*, not all functions.

(iii) As "formal" variables. For instance, we may talk about a formal polynomial $p(x) = 1 + x + x^2$. This is similar to sense (ii), but is not exactly the same. The variable x here is not necessarily a complex number, or in

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any fixed domain at all. It is a formal symbol, which we later replace by actual elements of the real numbers, or matrices, etc. at our whim. And p here is *not a function*; it is a polynomial.

The variables used in formal logic can also be considered to fall in sense (iii). For example, we may have a set of variables $\{x, y, z\}$ and a formula from the first-order language using such variables: $(\exists x((x \leq 0) \wedge \mathrm{R}(z))$.

(iv) As pieces of (experimental) data. Used in the sciences. One may say "at $t = 4\,\mathrm{s}$, $x = 23.1\,\mathrm{m}$" which may really mean: "at 4 seconds from the start of the experiment, the object is 23.1 metres to the right of its initial position".

So the symbols t and x are being used in the meaning of "time" and "position" in general. There may or may not be a functional relation between the "variables" t and x. If there is, we might say "x is a function of t", and we can talk about quantities such as dx/dt.

If we want to talk about a specific (but unnamed) time, we can use a notation such as "when $t = t_0, \ldots$" for some variable t_0 in sense (i).

The field of probability and statistics follows a similar practice for what are termed "random variables", which are really functions defined on a measure space Ω. But in practice they are usually denoted with variable notation: e.g. "the random variable X", and a specific value of this random variable X, at some unspecified $\omega \in \Omega$, is denoted by x.

(v) As state variables in computer algorithms. In this case, a variable x stands for a computer memory location. Or in more abstract language, x is a name for a container which may hold some object. The contents of this container may change as time passes or when it is modified by a program that the computer is executing.

In formal language, putting a value in the container is often denoted by notation like "$x \leftarrow 2$".

Note that the above distinctions are not always clear-cut. and the same symbol x may be used for different purposes at once, which of course, may lead to confusion.