square root of polynomial*

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2013-03-21 19:44:55

The *square root of a polynomial* f, denoted by $\sqrt{f}$, is any polynomial g having the square g^2 equal to f. For example, $\sqrt{9x^2-30x+25} = 3x-5$ or $-3x+5$.

A polynomial needs not have a square root, but if it has a square root g, then also the opposite polynomial $-g$ is its square root.

Algorithm. The idea of the squaring formula

$$(a+b+c+..)^2 = (a)a + (2a+b)b + (2a+2b+c)c + ..$$

(see the square of sum) gives a method for getting the square root of a polynomial:

- The terms of the radicand are ordered according to the rising or falling powers of certain letter (the first term must have a positive coefficient and even exponents).
- The leading term of the root is equal to the square root of the first term of the radicand.
- The second term of the root is equal to the first term of the first remainder divided by the double leading term.
- The third term of the root is equal to first term of the second remainder divided by the double leading term.
- And so on.

In the examples below, on the left under the lines there are the remainders, on the right under the lines the corresponding sums.

Example 1. $\sqrt{9x^4+6x^3-11x^2-4x+4} = ?$

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$$
\begin{array}{llllllllllll}
\sqrt{} & (9x^4 & +6x^3 & -11x^2 & -4x & +4) & = & \pm & (3x^2 & +x & -2) \\
 & \underline{9x^4} & & & & & & & \underline{3x^2} & & \\
 & & 6x^3 & -11x^2 & & & & & 6x^2 & +x & \\
 & & \underline{6x^3} & \underline{+x^2} & & & & & & \underline{x} & \\
 & & & -12x^2 & -4x & +4 & & & 6x^2 & +2x & -2 \\
 & & & \underline{-12x^2} & \underline{-4x} & \underline{+4} & & & & & \underline{-2} \\
 & & & & & 0 & & & & &
\end{array}
$$

Example 2. $\sqrt{x^6-2x^5-x^4+3x^2+2x+1}=\,?$

$$
\begin{array}{llllllllllllll}
\sqrt{} & (1 & +2x & +3x^2 & & -x^4 & -2x^5 & +x^6) & = & \pm & (1 & +x & +x^2 & -x^3) \\
 & \underline{1} & & & & & & & & & \underline{1} & & & \\
 & & 2x & +3x^2 & & & & & & & 2 & +x & & \\
 & & \underline{2x} & \underline{+x^2} & & & & & & & & \underline{+x} & & \\
 & & & 2x^2 & & -x^4 & & & & & 2 & +2x & +x^2 & \\
 & & & \underline{2x^2} & \underline{+2x^3} & \underline{+x^4} & & & & & & & \underline{x^2} & \\
 & & & & -2x^3 & -2x^4 & -2x^5 & +x^6 & & & 2 & +2x & +2x^2 & -x^3 \\
 & & & & \underline{-2x^3} & \underline{-2x^4} & \underline{-2x^5} & \underline{+x^6} & & & & & & \underline{-x^3} \\
 & & & & & & & 0 & & & & & &
\end{array}
$$

Remark. The procedure may give a Taylor series expansion of the square root, if it is not a polynomial. E.g. we get

$$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + - ...$$

References

[1] *Meyers Rechenduden.* Erster verbesserter Neudruck. Bibliographisches Institut AG, Mannheim (1960).