

proof of parallelogram law*

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The proof supplied here for the parallelogram law uses the properties of norms and inner products. See the entries about these objects for more details regarding the following calculations.

$$\begin{aligned} \|x + y\|^2 + \|x - y\|^2 &= \langle x + y, x + y \rangle + \langle x - y, x - y \rangle \\ &= \langle x, x + y \rangle + \langle y, x + y \rangle + \langle x, x - y \rangle - \langle y, x - y \rangle \\ &= \langle x + y, x \rangle + \langle x + y, y \rangle + \langle x - y, x \rangle - \langle x - y, y \rangle \\ &= \langle x, x \rangle + \langle y, x \rangle + \langle x, y \rangle + \langle y, y \rangle + \langle x, x \rangle - \langle y, x \rangle - \left(\langle x, y \rangle - \langle y, y \rangle \right) \\ &= \langle x, x \rangle + \langle y, x \rangle + \langle x, y \rangle + \langle y, y \rangle + \langle x, x \rangle - \langle y, x \rangle - \langle x, y \rangle + \langle y, y \rangle \\ &= \langle x, x \rangle + \langle y, y \rangle + \langle x, x \rangle + \langle y, y \rangle \\ &= 2\langle x, x \rangle + 2\langle y, y \rangle \\ &= 2\|x\|^2 + 2\|y\|^2. \end{aligned}$$

□

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