angle between two planes*

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Let π_1 and π_2 be two planes in the three-dimensional Euclidean space $\mathbb{R}^3$. The angle θ between these planes is defined by means of the normal vectors $\boldsymbol{n}_1$ and $\boldsymbol{n}_2$ of π_1 and π_2 through the relationship

$$\cos\theta = \left|\frac{\langle \boldsymbol{n}_1, \boldsymbol{n}_2\rangle}{\|\boldsymbol{n}_1\|\|\boldsymbol{n}_2\|}\right|,$$

where the numerator is the inner product of $\boldsymbol{n}_1$ and $\boldsymbol{n}_2$ and the denominator is product of the lengths of $\boldsymbol{n}_1$ and $\boldsymbol{n}_2$. The formula implies that the angle θ satisfies

$$0 \leq \theta \leq \frac{\pi}{2}.$$

The quotient in the formula remains unchanged as one multiplies the normal vectors by some non-zero real numbers, so that the cosine is independent of the lengths of the chosen vectors. Therefore, there is no ambiguity in this definition.

Generalization. The above definition can be generalized, at least locally, to a pair of intersecting differentiable surfaces in $\mathbb{R}^3$. Given two differentiable surfaces S_1 and S_2 and a point $p \in S_1 \cap S_2$, the angle between S_1 and S_2 at p is defined to be the angle between the tangent planes $T_p(S_1)$ and $T_p(S_2)$.

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Figure 1: Angle between two planes