

# consequence operator determined by a class of subsets\*

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**Theorem 1.** *Let  $L$  be a set and let  $K$  be a subset of  $\mathcal{P}(L)$ . The the mapping  $C: \mathcal{P}(L) \rightarrow \mathcal{P}(L)$  defined as  $C(X) = \cap\{Y \in K \mid X \subseteq Y\}$  is a consequence operator.*

*Proof.* We need to check that  $C$  satisfies the defining properties.

*Property 1:* Since every element of the set  $\{Y \in K \mid X \subseteq Y\}$  contains  $X$ , we have  $X \subseteq C(X)$ .

*Property 2:* For every element  $Y$  of  $K$  such that  $X \subseteq Y$ , it also is the case that  $C(X) \subseteq Y$  because an intersection of a family of sets is a subset of any member of the family. In other words (or rather, symbols),

$$\{Y \in K \mid X \subseteq Y\} \subseteq \{Y \in K \mid C(X) \subseteq Y\},$$

hence  $C(C(X)) \subseteq C(X)$ . By the first property proven above,  $C(X) \subseteq C(C(X))$  so  $C(C(X)) = C(X)$ . Thus,  $C \circ C = C$ .

*Property 3:* Let  $X$  and  $Y$  be two subsets of  $L$  such that  $X \subseteq Y$ . Then if, for some other subset  $Z$  of  $L$ , we have  $Y \subseteq Z$ , it follows that  $X \subseteq Z$ . Hence,

$$\{Z \in K \mid Y \subseteq Z\} \subseteq \{Z \in K \mid X \subseteq Z\},$$

so  $C(X) \subseteq C(Y)$ .

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\**(ConsequenceOperatorDeterminedByAClassOfSubsets)* created: *(2013-03-21)* by: *(rspuzio)* version: *(38671)* Privacy setting: *(1)* *(Theorem)* *(03G25)* *(03G10)* *(03B22)*

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