e is irrational*

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We have the series

$$e^{-1}=\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}$$

Note that this is an alternating series and that the magnitudes of the terms decrease. Hence, for every integer $n>0$, we have the bound

$$0<\left|\sum_{k=0}^{n}\frac{(-1)^k}{k!}-e^{-1}\right|<\frac{1}{(n+1)!},$$

by the Leibniz' estimate for alternating series. Assume that $e=n/m$, where m and n are integers and $n>0$. Then we would have

$$0<\left|\sum_{k=0}^{n}\frac{(-1)^k}{k!}-\frac{m}{n}\right|<\frac{1}{(n+1)!}.$$

Multiplying both sides by $n!$, this would imply

$$0<\left|\sum_{k=0}^{n}\frac{(-1)^k n!}{k!}-m(n-1)!\right|<\frac{1}{n+1},$$

which is a contradiction because every term in the sum is an integer, but there are no integers between 0 and $1/(n+1)$.

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