level curve*

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The *level curves* (in German *Niveaukurve*, in French *ligne de niveau*) of a surface

$$z = f(x, y) \tag{1}$$

in $\mathbb{R}^3$ are the intersection curves of the surface and the planes $z =$ constant. Thus the projections of the level curves on the xy-plane have equations of the form

$$f(x, y) = c \tag{2}$$

where c is a constant.

For example, the level curves of the hyperbolic paraboloid $z = xy$ are the rectangular hyperbolas $xy = c$ (cf. this entry).

The gradient $f'_x(x, y)\,\vec{i} + f'_y(x, y)\,\vec{j}$ of the function f in any point of the surface (1) is perpendicular to the level curve (2), since the slope of the gradient is $\frac{f'_y}{f'_x}$ and the slope of the level curve is $-\frac{f'_x}{f'_y}$, whence the slopes are opposite inverses.

Analogically one can define the *level surfaces* (or *contour surfaces*)

$$F(x, y, z) = c \tag{3}$$

for a function F of three variables x, y, z. The gradient of F in a point (x, y, z) is parallel to the surface normal of the level surface passing through this point.

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