trigonometric equations*

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A *trigonometric equation* contains values of given trigonometric functions whose arguments are unknown angles. The task is to determine all possible values of those angles. For obtaining the complete solution one needs the following properties of the trigonometric functions:

- Two angles have the same value of sine iff the angles are equal or supplementary angles or differ of each other by a multiple of full angle.
- Two angles have the same value of cosine iff the angles are equal or opposite angles or differ of each other by a multiple of full angle.
- Two angles have the same value of tangent iff the angles are equal or differ of each other by a multiple of straight angle.
- Two angles have the same value of cotangent iff the angles are equal or differ of each other by a multiple of straight angle.

The first principle in solving a trigonometric equation is that try to elaborate with goniometric formulae or else it so that only one trigonometric function on one angle remains in the equation. Then the equation is usually resolved to the form

$$f(kx) = a, \tag{1}$$

where k and a are known numbers and f is one of the functions sin, cos, tan, cot. Thereafter one can solve the values of the angle kx and, dividing these by k, at last the values of the angle x.

Example 1.

$$\sin x \cos x + \frac{1}{4} = 0$$

$$2 \sin x \cos x = -\frac{1}{2}$$

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$$\sin 2x = -\frac{1}{2}$$

$$\sin 2x = \sin 210°$$

$$2x = 210° + n \cdot 360° \quad \vee \quad 2x = 180° - 210° + n \cdot 360°$$

$$x = 105° + n \cdot 180° \quad \vee \quad x = -15° + n \cdot 180°$$

On the third line one used the double angle formula of sine.

It may happen that the form (1) cannot be attained, but instead e.g. the form

$$f(kx) = f(\alpha), \tag{2}$$

where x can be contained in α.

Example 2.

$$\sin 2x = \cos 3x$$

$$\cos(90° - 2x) = \cos 3x$$

$$90° - 2x = 3x + n \cdot 360° \quad \vee \quad 90° - 2x = -3x + n \cdot 360°$$

$$-2x - 3x = -90° + n \cdot 360° \quad \vee \quad -2x + 3x = -90° + n \cdot 360°$$

$$x = 18° + n \cdot 72° \quad \vee \quad x = -90° + n \cdot 360°$$

On the second line one of the complement formulas was utilized.

Example 3.

$$\sin 2x = -\sin 3x$$

$$\sin 2x = \sin(-3x)$$

$$2x = -3x + n \cdot 360° \quad \vee \quad 2x = 180° - (-3x) + n \cdot 360°$$

$$x = n \cdot 72° \quad \vee \quad x = 180° + n \cdot 360°$$

On the second line the opposite angle formula of sine was utilized.

Example 4.

$$\cos 2x = -\cos 3x$$

$$\cos 2x = \cos(180° - 3x)$$

$$2x = \pm(180° - 3x) + n \cdot 360°$$

$$x = 36° + n \cdot 72° \quad \vee \quad x = 180° + n \cdot 360°$$

On the second line the supplement formula of sine was utilized.