

a characterization of the radical of an ideal*

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2013-03-22 1:03:57

Proposition 1. *Let I be an ideal in a ring R , and \sqrt{I} be its radical. Then \sqrt{I} is the intersection of all prime ideals containing I .*

Proof. Suppose $x \in \sqrt{I}$, and P is a prime ideal containing I . Then $R - P$ is an m -system. If $x \in R - P$, then $(R - P) \cap I \neq \emptyset$, contradicting the assumption that $I \subseteq P$. Therefore $x \notin R - P$. In other words, $x \in P$, and we have one of the inclusions.

Conversely, suppose $x \notin \sqrt{I}$. Then there is an m -system S containing x such that $S \cap I = \emptyset$. Enlarge I to a prime ideal P disjoint from S , so that $x \notin P$ (we can do this; for a proof, see the second remark in this entry). By contraposition, we have the other inclusion. \square

Remark. This shows that every prime ideal is a radical ideal: for \sqrt{P} is the intersection of all prime ideals containing P , and if P is itself prime, then $P = \sqrt{P}$.

**ACharacterizationOfTheRadicalOfAnIdeal* created: $\langle 2013-03-2 \rangle$ by: $\langle CWoo \rangle$ version: $\langle 40617 \rangle$ Privacy setting: $\langle 1 \rangle$ $\langle Derivation \rangle$ $\langle 16N40 \rangle$ $\langle 13-00 \rangle$ $\langle 14A05 \rangle$

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