table of generalized Fourier and measured groupoid transforms*

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0.1 Generalized Fourier transforms

Fourier-Stieltjes transforms and **measured groupoid** transforms are useful generalizations of the (much simpler) Fourier transform, as concisely shown in the following table- with the same format as C. Woo's Feature on Fourier transforms - for the purpose of direct comparison with the latter transform. Unlike the more general Fourier-Stieltjes transform, the Fourier transform exists if and only if the function to be transformed is Lebesgue integrable over the whole real axis for $t \in \mathbb{R}$, or over the entire $\mathbb{C}$ domain when $\check{m}(t)$ is a complex function.

Definition 0.1. Fourier-Stieltjes transform.

Given a *positive definite, measurable function* $f(x)$ on the interval $(-\infty, \infty)$ there exists a monotone increasing, real-valued bounded function $\alpha(t)$ such that:

$$f(x) = \int_{\mathbb{R}} e^{itx} d(\alpha(t), \tag{0.1}$$

for all $x \in \mathbb{R}$ except a small set. When $f(x)$ is defined as above and if $\alpha(t)$ is nondecreasing and bounded then the measurable function defined by the above integral is called *the Fourier-Stieltjes transform of* $\alpha(t)$, and it is continuous in addition to being positive definite.

*⟨TableOfGeneralizedFourierAndMeasuredGroupoidTransforms⟩ created: ⟨2013-03-2⟩ by: ⟨bci1⟩ version: ⟨40739⟩ Privacy setting: ⟨1⟩ ⟨Topic⟩ ⟨55U99⟩

FT Generalizations

$f(t)$	$\mathcal{F}f(t) = \hat{f}(x)$	Conditions*	Explanation	Description
c	$(\sqrt{2\pi})^{-1}c$	Notice on the next line the overline bar placed above $t(x)$		
$f(t)$	$\int \hat{f}(x)\overline{t(x)}dx$	$f(t) \in L^1(G_l)$, with G_l a locally compact groupoid [**?**]; $\int$ is defined *via* a left Haar measure on G_l	Fourier-Stieltjes transform	$\hat{f}(x) \in C_0(\hat{G}_l)$
$\hat{m}(x)$	$\check{m}(t) = \int e^{itx} d\hat{m}(x)$	as above	Inverse Fourier-Stieltjes transform	$\check{m}(t) \in L^1(G_l)$, ([**?**], [**?**]).
$\hat{m}(x)$	$\check{m}(t) = \int e^{itx} d\hat{m}(x)$	When $G_l = \mathbb{R}$, and it exists only when $\hat{m}(x)$ is *Lebesgue integrable* on the entire real axis	This is the usual Inverse Fourier transform	$\check{m}(t) \in \mathbb{R}$

*Note the 'slash hat' on $\hat{f}(x)$ and $\hat{G}_l$.

References

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