non-Abelian theory*

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Definition 0.1. A *non-Abelian theory* is one that does not satisfy one, several, or all of the axioms of an Abelian theory, such as, for example, those for an Abelian category theory.

0.1 Examples

ETAC and ETAS axiom interpretations that do not satisfy–in addition to the ETAC or ETAS axioms– the $Ab1$ to $Ab6$ axioms for an abelian category are all examples on non-Abelian categories; a more detailed list is also presented next.

Remark 0.1. In a general sense, any Abelian category (or *abelian category*) can be regarded as a 'good' model for the category of Abelian, or commutative, groups. Furthermore, in an Abelian category Ab every class, or set, of morphisms $Hom_{Ab}(-,-)$ forms an Abelian (or commutative) group. There are several strict definitions of Abelian categories involving 3, 4 or up to 6 axioms defining the Abelian character of a category. To illustrate non-Abelian theories it is useful to consider non-Abelian structures so that specific properties determined by the non-Abelian set of axioms become 'transparent' in terms of the properties of objects for example for concrete categories that have objects; such examples are presented separately as *non-Abelian structures*.

0.2 Further examples of non-Abelian theories

The following is only a short list of non-Abelian theories:

1. Non-Abelian algebraic topology, including also non-Abelian homological algebra; non-Abelian algebraic topology overview and R. Brown 2008 preprint, ([**?**, **?**]).
   (See also the recent book exposition with the title *"Nonabelian Algebraic Topology"* vol. 1 by Brown and Sivera,(respectively, vol. 2 with Higgins, *in preparation*).
2. Non-Abelian quantum algebraic topology;
3. Non-Abelian gauge field theory (in Quantum Physics);
4. Noncommutative geometry;
5. The axiomatic theory of supercategories (ETAS);
6. Higher dimensional algebra (HDA)
7. LM_n Logic algebras;
8. Non-Abelian categorical ontology ([**?**]).

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0.3 Remarks

The following alternative definition by Barry Mitchell of an Abelian category should also be mentioned as *"an exact additive category with finite products."*.

He also published in his textbook the following theorem: (**Theorem 20.1**, on p.33 of Barry Mitchell in "Theory of Catgeories", 1965, Academic Press: New York and London):

Theorem 0.1. *"The following statements are equivalent:*

- *(a) Ab is an abelian category;*
- *(b) Ab has kernels, cokernels, finite products, finite coproducts, and is both normal and conormal;*
- *(c) Ab has pushouts and pullbacks and is both normal and conormal."*

References

[1] R. Brown et al. 2008. *"Non-Abelian Algebraic Topology"*. vols. 1 and 2. (*Preprint*).

[2] R. Brown. 2008. *Higher Dimensional Algebra Preprint as pdf and ps docs. at* $arXiv : math/0212274v6[math.AT]$

[3] I. C. Baiantu, R. Brown and J. F. Glazebrook. 2007, A Non–Abelian Categorical Ontology and Higher Dimensional Algebra of Spacetimes and Quantum Gravity., *Axiomathes* , **17**: 353-408.