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metric superfields*

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This is a topic entry on metric superfields in quantum supergravity and the mathematical cncepts related to spinor and tensor fields.

1 Metric superfields: spinor and tensor fields

Because in supergravity both spinor and tensor fields are being considered, the gravitational fields are represented in terms of *tetrads*, $e^a_\mu(x)$, rather than in terms of the general relativistic metric $g_{\mu\nu}(x)$. The connections between these two distinct representations are as follows:

$$g_{\mu\nu}(x) = \eta_{ab}\ e^a_\mu(x) e^b_\gamma(x)\ , \tag{1.1}$$

with the general coordinates being indexed by μ, ν, etc., whereas local coordinates that are being defined in a locally inertial coordinate system are labeled with superscripts a, b, etc.; η_{ab} is the diagonal matrix with elements +1, +1, +1 and -1. The tetrads are invariant to two distinct types of symmetry transformations–the local Lorentz transformations:

$$e^a_\mu(x) \longmapsto \Lambda^a_b(x) e^b_\mu(x)\ , \tag{1.2}$$

(where Λ^a_b is an arbitrary real matrix), and the general coordinate transformations:

$$x^\mu \longmapsto (x')^\mu(x)\ . \tag{1.3}$$

In a weak gravitational field the tetrad may be represented as:

$$e^a_\mu(x) = \delta^a_\mu(x) + 2\kappa\Phi^a_\mu(x)\ , \tag{1.4}$$

where $\Phi^a_\mu(x)$ is small compared with $\delta^a_\mu(x)$ for all x values, and $\kappa = \sqrt{8\pi G}$, where G is Newton's gravitational constant. As it will be discussed next, the supersymmetry algebra (SA) implies that the graviton has a fermionic superpartner, the hypothetical *gravitino*, with helicities ± 3/2. Such a self-charge-conjugate massless particle as the gravitiono with helicities ± 3/2 can only have *low-energy* interactions if it is represented by a Majorana field $\psi_\mu(x)$ which is invariant under the gauge transformations:

$$\psi_\mu(x) \longmapsto \psi_\mu(x) + \delta_\mu\psi(x)\ , \tag{1.5}$$

with $\psi(x)$ being an arbitrary Majorana field as defined by Grisaru and Pendleton (1977). The tetrad field $\Phi_{\mu\nu}(x)$ and the graviton field $\psi_\mu(x)$ are then incorporated into a term $H_\mu(x,\theta)$ defined as the *metric superfield.* The relationships between $\Phi_{\mu_\nu}(x)$ and $\psi_\mu(x)$, on the one hand, and the components of the metric superfield $H_\mu(x,\theta)$, on the other hand, can be derived from the transformations of the whole metric superfield:

$$H_\mu(x,\theta) \longmapsto H_\mu(x,\theta) + \Delta_\mu(x,\theta)\ , \tag{1.6}$$

by making the simplifying– and physically realistic– assumption of a weak gravitational field (further details can be found, for example, in Ch.31 of vol.3. of Weinberg, 1995). The interactions of the entire superfield $H_\mu(x)$ with matter would be then described by considering how a weak gravitational field, h_{μ_ν} interacts with an energy-momentum tensor

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$T^{\mu\nu}$ represented as a linear combination of components of a real vector superfield Θ^{μ}. Such interaction terms would, therefore, have the form:

$$I_{\mathcal{M}} = 2\kappa \int dx^4 [H_{\mu}\Theta^{\mu}]_D \ , \tag{1.7}$$

($\mathcal{M}$ denotes 'matter') integrated over a four-dimensional (Minkowski) spacetime with the metric defined by the superfield $H_{\mu}(x, \theta)$. The term Θ^{μ}, as defined above, is physically a *supercurrent* and satisfies the conservation conditions:

$$\gamma^{\mu}\mathbf{D}\Theta_{\mu} = \mathbf{D} \ , \tag{1.8}$$

where $\mathbf{D}$ is the four-component super-derivative and X denotes a real chiral scalar superfield. This leads immediately to the calculation of the interactions of matter with a weak gravitational field as:

$$I_{\mathcal{M}} = \kappa \int d^4x T^{\mu\nu}(x) h_{\mu\nu}(x) \ , \tag{1.9}$$

It is interesting to note that the gravitational actions for the superfield that are invariant under the generalized gauge transformations $H_{\mu} \longmapsto H_{\mu} + \Delta_{\mu}$ lead to solutions of the Einstein field equations for a homogeneous, non-zero vacuum energy density ρ_V that correspond to either a de Sitter space for $\rho_V > 0$, or an anti-de Sitter space for $\rho_V < 0$. Such spaces can be represented in terms of the hypersurface equation

$$x_5^2 \pm \eta_{\mu,\nu} x^{\mu} x^{\nu} = R^2 \ , \tag{1.10}$$

in a quasi-Euclidean five-dimensional space with the metric specified as:

$$ds^2 = \eta_{\mu,\nu} x^{\mu} x^{\nu} \pm dx_5^2 \ , \tag{1.11}$$

with '+' for de Sitter space and '-' for anti-de Sitter space, respectively.

Note The presentation above follows the exposition by S. Weinberg in his book on "Quantum Field Theory" (2000), vol. 3, Cambridge University Press (UK), in terms of both concepts and mathematical notations.