0.1 Duality in mathematics

The following is a mathematical topic entry on different types of duality encountered in different areas of mathematics; accordingly there is a string of distinct definitions associated with this topic rather than a single, general definition, although some of the linked definitions, that is, categorical duality, are more general than others.

0.1.1 Duality definitions in mathematics:

1. Categorical duality and Dual category: reversing arrows
2. Duality principle
3. Double duality
4. Triality
5. Self-duality
6. Duality functors, (for example the duality functor $\text{Hom}_k(\_, k)$)
7. Poincaré duality/Poincaré isomorphism
8. Poincaré-Lefschetz duality, and Alexander-Lefschetz duality
10. Serre duality: example- in the proof of the Riemann-Roch theorem for curves
11. Dualities in logic, example: De Morgan dual, Boolean algebra
12. Stone duality: Boolean algebras and Stone spaces
13. Dual numbers- as in an associative algebra; (almost synonymous with double)

14. Geometric dualities: dual polyhedron, dual of a planar graph, duality in order theory, the Legendre transformation -an application of the duality between points and lines; generalized Legendre, that is, the Legendre-Fenchel transformation.

15. Hamilton–Lagrange duality in theoretical mechanics and optics

16. **Dual space**

17. **Dual space example**

18. **Dual homomorphisms**

19. **Duality of Projective Geometry**

20. Analytic dualities

21. **Duals of an algebra/algebraic duality**, for example, dual pairs of Hopf *-algebras and duality of cross products of C*-algebras

22. **Tangled, or Mirror, duality** interchanging morphisms and objects

23. Duality as a homological mirror symmetry

24. Cohomology theory duals: de Rham cohomology ↔ Alexander-Spanier cohomology

25. Hodge dual

26. **Duality of locally compact groups**

27. **Pontryagin duality**, for locally compact commutative topological groups and their linear representations

28. **Tannaka-Krein duality** for compact matrix pseudogroups and non-commutative topological groups; its generalization leads to quantum groups in Quantum theories; Tannaka’s theorem provides the means to reconstruct a compact group $G$ from its category of representations $\Pi(G)$; Krein's theorem shows which categories arise as a dual object to a compact group; the finite-dimensional representations of Drinfel’d ’s quantum groups form a braided monoidal category, whereas $\Pi(G)$ is a symmetric monoidal category.

29. Tannaka duality: an extension of Tannakian duality by Alexander Grothendieck to algebraic groups and Tannakian categories.

30. Contravariant dualities
31. Weak duality, example: weak duality theorem in linear programming; dual problems in optimization theory

32. Dual codes

33. Duality in Electrical Engineering

0.1.2 Examples of duals:

1. a category $\mathcal{C}$ and its dual $\mathcal{C}^{op}$

2. the category of Hopf algebras over a field is (equivalent to) the opposite category of affine group schemes over $\text{spec } k$

3. Dual Abelian variety

4. Example of a dual space theorem

5. Example of Pontryagin duality

6. initial and final object

7. kernel and cokernel

8. limit and colimit

9. direct sum and product

References
