

# Laplace transform of integral\*

*pahio*<sup>†</sup>

2013-03-22 1:47:09

One can show that if a real function  $t \mapsto f(t)$  is Laplace-transformable, as well is  $\int_0^t f(\tau) d\tau$ . The latter is also continuous for  $t > 0$  and by the Newton–Leibniz formula, has the derivative equal  $f(t)$ . Hence we may apply the formula for Laplace transform of derivative, obtaining

$$F(s) = \mathcal{L}\{f(t)\} = s \mathcal{L}\left\{\int_0^t f(\tau) d\tau\right\} - \int_0^0 f(t) dt = s \mathcal{L}\left\{\int_0^t f(\tau) d\tau\right\},$$

i.e.

$$\mathcal{L}\left\{\int_0^t f(\tau) d\tau\right\} = \frac{F(s)}{s}. \quad (1)$$

**Application.** We start from the easily derivable rule

$$\frac{1}{s} \curvearrowright 1,$$

where the curved arrow points from the Laplace-transformed function to the original function. The formula (1) thus yields successively

$$\begin{aligned} \frac{1}{s^2} &\curvearrowright \int_0^t 1 d\tau = t, \\ \frac{1}{s^3} &\curvearrowright \int_0^t \tau d\tau = \frac{t^2}{2!}, \\ \frac{1}{s^4} &\curvearrowright \int_0^t \frac{\tau^2}{2!} d\tau = \frac{t^3}{3!}, \end{aligned}$$

etc. Generally, one has

$$\frac{1}{s^n} \curvearrowright \frac{t^{n-1}}{(n-1)!} \quad \forall n \in \mathbb{Z}_+. \quad (2)$$

---

\**(LaplaceTransformOfIntegral)* created: *(2013-03-2)* by: *(pahio)* version: *(41081)* Privacy setting: *(1)* *(Derivation)* *(44A10)*

<sup>†</sup>This text is available under the Creative Commons Attribution/Share-Alike License 3.0. You can reuse this document or portions thereof only if you do so under terms that are compatible with the CC-BY-SA license.