

lengths of angle bisectors*

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In any triangle, the lengths w_a , w_b , w_c of the angle bisectors opposing the sides a , b , c , respectively, are

$$w_a = \frac{\sqrt{bc[(b+c)^2 - a^2]}}{b+c}, \quad (1)$$

$$w_b = \frac{\sqrt{ca}[(c+a)^2 - b^2]}{c+a}, \quad (2)$$

$$w_c = \frac{\sqrt{ab}[(a+b)^2 - c^2]}{a+b}. \quad (3)$$

Proof. By the symmetry, it suffices to prove only (1).

According to the angle bisector theorem, the bisector w_a divides the side a into the portions

$$\frac{b}{b+c} \cdot a = \frac{ab}{b+c}, \quad \frac{c}{b+c} \cdot a = \frac{ca}{b+c}.$$

If the angle opposite to a is α , we apply the law of cosines to the half-triangles separated by w_a :

$$\begin{cases} 2w_a b \cos \frac{\alpha}{2} = w_a^2 + b^2 - \left(\frac{ab}{b+c}\right)^2 \\ 2w_a c \cos \frac{\alpha}{2} = w_a^2 + c^2 - \left(\frac{ca}{b+c}\right)^2 \end{cases} \quad (4)$$

For eliminating the angle α , the equations (4) are divided sidewise, when one gets

$$\frac{b}{c} = \frac{w_a^2 + b^2 - \left(\frac{ab}{b+c}\right)^2}{w_a^2 + c^2 - \left(\frac{ca}{b+c}\right)^2},$$

from which one can after some routine manipulations solve w_a , and this can be simplified to the form (1).

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