properties of regular tetrahedron[*]

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2013-03-22 1:55:44

A regular tetrahedron may be formed such that each of its edges is a diagonal of a face of a cube; then the tetrahedron has been inscribed in the cube.

It's apparent that a plane passing through the midpoints of three parallel edges of the cube cuts the regular tetrahedron into two congruent pentahedrons and that the intersection figure is a square, the midpoint M of which is the centroid of the tetrahedron.

The angles between the four half-lines from the centroid M of the regular tetrahedron to the vertices are $2\arctan\sqrt{2}$ ($\approx 109^\circ$), which is equal the angle between the four covalent bonds of a carbon atom. A half of this angle, α, can be found from the right triangle in the below figure, where the catheti are $\frac{s}{\sqrt{2}}$ and $\frac{s}{2}$.

[*]⟨*PropertiesOfRegularTetrahedron*⟩ created: ⟨*2013-03-2*⟩ by: ⟨*pahio*⟩ version: ⟨*41178*⟩ Privacy setting: ⟨*1*⟩ ⟨*Topic*⟩ ⟨*51E99*⟩

$\frac{s}{\sqrt{2}}$

M α s

s

s

One can consider the regular tetrahedron as a cone. Let its edge be a and its height h. Because of symmetry, a height line intersects the corresponding base triangle in the centroid of this equilateral triangle. Thus we have (see the below diagram) the rectangular triangle with hypotenuse a, one cathetus h and the other cathetus $\frac{2}{3}\cdot\frac{a\sqrt{3}}{2} = \frac{a}{\sqrt{3}}$ (i.e. $\frac{2}{3}$ of the median $\frac{a\sqrt{3}}{2}$ of the equilateral triangle — see the common point of triangle medians). The Pythagorean theorem then gives

$$h = \sqrt{a^2 - \left(\frac{a}{\sqrt{3}}\right)^2} = \frac{a\sqrt{6}}{3}.$$

a

h

$\frac{a}{2}$

$\frac{a}{2}$

Consequently, the height of the regular tetrahedron is $\dfrac{a\sqrt{6}}{3}$.

Since the area of the base triangle is $\frac{a^2\sqrt{3}}{4}$, the volume (one third of the product of the base and the height) of the regular tetrahedron is $\dfrac{a^3\sqrt{2}}{12}$.