Non-Newtonian Calculus*

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0.1 Brief Description

The non-Newtonian calculi provide a wide variety of mathematical tools for use in science, engineering, and mathematics. They appear to have considerable potential for use as alternatives to the classical calculus of Newton and Leibniz.

There are infinitely many non-Newtonian calculi. Like the classical calculus, each of them possesses (among other things): a derivative, an integral, a natural average, a special class of functions having a constant derivative, and two Fundamental Theorems which reveal that the derivative and integral are 'inversely' related. Nevertheless, many non-Newtonian calculi are markedly different from the classical calculus.

For example, infinitely many non-Newtonian calculi have a nonlinear derivative or integral. Among these calculi are the geometric calculus, bigeometric calculus, harmonic calculus, biharmonic calculus, quadratic calculus, and biquadratic calculus. Furthermore, in the geometric calculus and in the bigeometric calculus, the derivative and integral are both multiplicative. (Please see the "Multiplicative Calculus" section of this website.)

Of course in the classical calculus, the linear functions are the functions having a constant derivative. However, in the geometric calculus, the exponential functions are the functions having a constant derivative. And in the bigeometric calculus, the power functions are the functions having a constant derivative. (The geometric derivative and the bigeometric derivative are closely related to the well-known logarithmic derivative and elasticity, respectively.)

The well-known arithmetic average (of functions) is the natural average in the classical calculus, but the well-known geometric average is the natural average in the geometric calculus. And the well-known harmonic average and quadratic average (or root mean square) are closely related to the natural averages in the harmonic and quadratic calculi, respectively.

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Furthermore, unlike the classical derivative, the bigeometric derivative is scale invariant (or scale free), i.e., it is invariant under all changes of scale (or unit) in function arguments and values.

Non-Newtonian calculus has application in several areas of science, engineering, and mathematics. Included among those areas are image analysis (e.g., in bio-medicine), growth and decay, rates of return, the theory of elasticity in economics, the neoclassical (Solow-Swan) exogenous growth model in economics, finance, the economics of climate change, atmospheric temperature (e.g., optical measure theory and inverse transfer theory),information technology, biology, materials science, pathogen counts in treated water, actuarial science, demographics, differential equations (including a multiplicative Lorenz system), calculus of variations, finite-difference methods, averages of functions, means of two positive numbers, weighted calculus and meta-calculus, multiplicative calculus for complex analysis, functional analysis, probability theory, utility theory, Bayesian analysis, stochastics, decision making, dynamical systems, chaos theory, fractals, and dimensional spaces. For more details, please see the Applications section of this website.

It is natural to speculate about future applications of non-Newtonian calculus, and of related matters such as weighted calculus, meta-calculus, averages, and means. Perhaps scientists, engineers, and mathematicians will use them to define new concepts, to yield new or simpler laws, or to formulate or solve problems.

0.2 "Multiplicative Calculus"

The geometric calculus and the bigeometric calculus are among the infinitely many multiplicative non-Newtonian calculi, i.e., non-Newtonian calculi in which the derivative and integral are both multiplicative. Accordingly, the expression "the multiplicative calculus" should be avoided, and no one specific calculus should be named "multiplicative calculus".

Some authors have used the name "multiplicative calculus" for the geometric calculus, while others have used the same name for the bigeometric calculus. It is hoped that the scientific community will soon reach accord with regard to names for these two calculi. Our suggestion is simply to use the names "geometric calculus" and "bigeometric calculus", respectively.

0.3 History

The non-Newtonian calculi were created in the period from 1967 to 1970 by Michael Grossman and Robert Katz. In July of 1967, they created an infinite family of calculi that includes the classical calculus, the geometric calculus, the harmonic calculus, and the quadratic calculus. In August of 1970, they created infinitely-many other calculi, including the bigeometric calculus, the biharmonic calculus, and the biquadratic calculus. All of the calculi can be described simultaneously within the framework of a general theory. They decided to use the adjective ”non-Newtonian” to indicate any of the calculi other than the classical calculus.

In 1972, Grossman and Katz completed their book Non-Newtonian Calculus. (Please see [15] in the References.) It contains discussions of nine specific non-Newtonian calculi (including the six previously mentioned), the general theory of non-Newtonian calculus, and heuristic guides for application. Subsequently, with Jane Grossman, they wrote several other books/articles on non-Newtonian calculus, and on related matters such as weighted calculus, meta-calculus, averages, and means. (Please see items [7 - 15, 34, 35] in the References section below.)

Michael Grossman and Robert Katz began their development of non-Newtonian calculus on 14 July 1967. In ”Non-Newtonian Calculus” (1972), they included the following paragraph (page 82): ”However, since we have nowhere seen a discussion of even one specific non-Newtonian calculus, and since we have not found a notion that encompasses the *-average, we are inclined to the view that the non-Newtonian calculi have not been known and recognized heretofore. But only the mathematical community can decide that.” In fact, prior to 14 July 1967, Grossman, Grossman, and Katz knew nothing about non-Newtonian calculus.

NOTE. The six books by Grossman, Grossman, and Katz on non-Newtonian calculus and related matters are indicated below, and are available at some academic libraries, public libraries, and book stores such as Amazon.com. On the Internet, each of the books can be read (free of charge) at Google Books, and each of them can be read and/or downloaded (free of charge) at HathiTrust. (1) Michael Grossman and Robert Katz: ”Non-Newtonian Calculus”, ISBN 0912938013, 1972. [15] (2) Michael Grossman: ”The First Nonlinear System of Differential and Integral Calculus”, ISBN 0977117006, 1979. [11] (3) Jane Grossman, Michael Grossman, Robert Katz: ”The First Systems of Weighted Differential and Integral Calculus”, ISBN 0977117014, 1980. [9] (4) Jane Grossman: ”Meta-Calculus: Differential and Integral”, ISBN 0977117022, 1981. [7] (5) Michael Grossman: ”Bigeometric Calculus: A System with a Scale-Free Derivative”, ISBN 0977117030, 1983. [10] (6) Jane Grossman, Michael Grossman, and Robert Katz: ”Averages: A New Approach”, ISBN 0977117049, 1983. [8]

0.4 Applications

Non-Newtonian calculus was used by James R. Meginniss (Claremont Graduate School and Harvey Mudd College) to create a theory of probability that is adapted to human behavior and decision making. (Please see [16] in the References.)

Non-Newtonian calculus was used by Diana Andrada Filip (Babes-Bolyai University of Cluj-Napoca, Romania) and Cyrille Piatecki (LEO, Orleans University, France) to re-postulate and analyse the neoclassical (Solow-Swan) exogenous growth model in economics. [82, 121] In their article [82] they state: ”In this paper, we have tried to present how a non-Newtonian calculus could be applied to repostulate and analyse the neoclassical [Solow-Swan] exogenous growth model [in economics]. ... In fact, one must acknowledge that it is only under the effort of Grossman and Katz (1972) ... that such a non-Newtonian calculus emerged to give a natural answer to many growth phenomena. ... We must underscore that to discover that there was a non-Newtonian way to look to differential equations has been a great surprise for us. It opens the question to know if there are major fields of economic analysis which can be profoundly re-thought in the light of this discovery."

Non-Newtonian calculus was used in the study of biomedical image analysis by Luc Florack and Hans van Assen (both of the Eindhoven University of Technology in the Netherlands). Their work has application to "complex imaging frameworks, such as diffusion tensor imaging". In their article [88], they state: "We advocate the use of an alternative calculus in biomedical image analysis, known as multiplicative (a.k.a. non-Newtonian) calculus." (The "multiplicative calculus" referred to here is the geometric calculus.) [88, 96, 111]

A course on non-Newtonian calculus was conducted in the summer-term of 2012 by Joachim Weickert, Laurent Hoeltgen, and other faculty from the Mathematical Image Analysis Group of Saarland University in Germany. Among the topics covered were applications of non-Newtonian calculus to digital image processing, rates of return, and other growth processes. [106]

Non-Newtonian calculus was used in the study of contour detection in images with multiplicative noise by Marco Mora, Fernando Cordova-Lepe, and Rodrigo Del-Valle (all of Universidad Catolica del Maule in Chile). [99]

Application of the bigeometric derivative to the theory of elasticity in economics was made by Fernando Cordova-Lepe (Universidad Catolica del Maule in Chile) . (He referred to the bigeometric derivative as the "multiplicative derivative.") [3, 4, 105] Elasticity and its relationship to the bigeometric derivative is also discussed in Non-Newtonian Calculus [15] and Bigeometric Calculus: A System with a Scale-Free Derivative [10].

Several applications of non-Newtonian calculus were made by Agamirza E. Bashirov, Mustafa Riza, and Yucel Tandogdu (all of Eastern Mediterranean University in Cyprus); Emine Misirli Kurpinar and Yusuf Gurefe (both of Ege University in Turkey); and Ali Ozyapici (Lefke European University in Turkey). Their work has application to differential equations, calculus of variations, finite-difference methods, complex analysis, actuarial science, finance, economics, biology, and demographics. [2, 24, 27, 33, 84, 87, 94, 95, 123, 140] (The article [2] was "submitted by Steven G. Krantz" and published in 2008 by the Journal of Mathematical Analysis and Applications.)

Non-Newtonian calculus was used by Ali Uzer (Fatih University in Turkey) to develop a multiplicative type of calculus for complex-valued functions of a complex variable. [78]

Application of non-Newtonian calculus to functional analysis was made by Cengiz Turkmen and Feyzi Basar, both of Fatih University in Turkey. Their work was presented at the First International Conference on Analysis and Applied Mathematics, whose purpose was "to bring together mathematicians working in the area of analysis and applied mathematics to share new trends of applications of math". [112]

Non-Newtonian calculus was used by Ahmet Faruk Cakmak (Yildiz Technical University in Turkey) and Feyzi Basar (Fatih University in Turkey) to yield "some new results on sequence spaces with respect to non-Newtonian calculus". [122]

The mathematics department of Eastern Mediterranean University in Cyprus has established a research group for the purpose of studying and applying the geometric calculus and the bigeometric calculus. [110]

Knowledge of the geometric calculus ("multiplicative calculus") is a requirement for the master's degree in computer-engineering at Inonu University (Malatya, Turkey). [136]

Seminars concerning non-Newtonian calculus and the dynamics of random fractal structures were conducted by Wojbor Woycznski (Case Western Reserve University) at The Ohio State University on 22 April 2011, and at Cleveland State University on 02 May 2012. [90, 104]

The bigeometric calculus was used in an article [126] on chaos in multiplicative dynamical systems by Dorota Aniszewska and Marek Rybaczuk, both from the Wroclaw University of Technology in Poland. They proved that "all classical conditions concerning chaotic behavior can be extended to multiplicative [dynamical] systems". Their work involves one-dimensional multiplicative versions of logistic equations, and multi-dimensional nonlinear dynamical systems described by means of the bigeometric derivative. The multiplicative version of the classical Lorenz system (as well as the Lyapunov exponent and the multiplicative Runge-Kutta method) was used "for analysis of stability and chaotic behavior".

The bigeometric derivative was used to reformulate the Volterra product integral. [1, 21, 22]

The bigeometric calculus was used in an article on multiplicative differential equations by Dorota Aniszewska (Wroclaw University of Technology). [129]

The bigeometric calculus was used in an article on a multiplicative Lorenz system by Dorota Aniszewska and Marek Rybaczuk (both from Wroclaw University of Technology). [130]

The bigeometric calculus was used in an article on multiplicative dynamical systems by Dorota Aniszewska and Marek Rybaczuk (both from Wroclaw University of Technology). [131]

The bigeometric calculus was used in an article on fractals and material science by M. Rybaczuk and P. Stoppel (both from Wroclaw University of Technology). [18]

The bigeometric calculus was used in an article on fractal dimension and dimensional spaces by Marek Rybaczuka (Wroclaw University of Technology in Poland), Alicja Kedziab (Medical Academy of Wroclaw in Poland), and Witold Zielinskia (Wroclaw University of Technology). [132]

According to [21], "in dimensional spaces (in a similar way to physical quantities) you can multiply and divide quantities which have different dimensions but you cannot add and subtract quantities with different dimensions. This means that the classical additive derivative is undefined because the difference f(x+deltax)-f(x) has no value. However in dimensional spaces, the geometric derivative and the bigeometric derivative remain well-defined. Multiplicative dynamical systems can become chaotic even when the corresponding classical additive system does not because the additive and multiplicative derivatives become inequivalent if the variables involved also have a varying fractal dimension."

Application of non-Newtonian calculus to information technology was made by S. L. Blyumin of the Lipetsk State Technical University in Russia. [23]

Applications of non-Newtonian calculus and related matters to various subjects are discussed in [15, 11, 9, 7, 10, 8, 12, 14, 34, 35]. Included among those subjects are analytic geometry, vectors, least-squares methods, centroids, complex numbers, sigmoidal functions, relativistic composition of speeds, measurement (physics), psychophysics, weighted calculus, meta-calculus, averages (of functions), and means (of two positive numbers).

The non-Newtonian natural averages (of functions) were used to construct a family of means (of two positive numbers). [8, 14] Included among those means are some well-known ones such as the arithmetic mean, the geometric mean, the harmonic mean, the power means, the logarithmic mean, the identric mean, and the Stolarsky mean. The family of means was used to yield simple proofs of some familiar inequalities. [14] Publications [8, 14] about that family are cited in articles [29-32, 118].

Non-Newtonian calculus was used in the famous 2006 report "Stern Review on the Economics of Climate Change", according to a 2012 critique of that report (called "What is Wrong with Stern?") by former UK Cabinet Minister Peter Lilley and economist Richard Tol. "Stern Review on the Economics of Climate Change", which is over 700-pages long, was commissioned by the UK government, was written by a team led by Nicholas Stern (former Chief Economist at the World Bank), and has drawn worldwide attention. [116]

Application of non-Newtonian calculus to the study of pathogen counts in treated water was made by James D. Englehardt (University of Miami) and Ruochen Li (Shenzhen, China). [85]

Weighted non-Newtonian calculus [9] was used by Ziyue Liu and Wensheng Guo (both of the University of Pennsylvania) in an article on spline smoothing. [119]

Weighted non-Newtonian calculus [9] was used by David Baqaee (Harvard University) in an article on an axiomatic foundation for intertemporal decision making. [86]

Non-Newtonian calculus was used by Stanley Paul Palasek (Sonoran Science Academy in Tucson, Arizona) in a biology project on opioid peptide delivery at an Intel International Science and Engineering Fair. [97]

Non-Newtonian calculus was used by J. I. King in a study on atmospheric temperature (optical measure theory and inverse transfer theory). [102] (This study is used in [89].)

A lecture about the bigeometric calculus was presented by Ahmet Faruk Cakmak at the 2011 International Conference on Applied Analysis and Algebra at Yildiz Technical University in Istanbul, Turkey. [107]

The geometric calculus was used by Ugur Kadak (Gazi University in Turkey) and Yusuf Gurefe (Bozok University in Turkey) in their presentation at the 2012 Analysis and Applied Mathematics Seminar Series of Fatih University in Istanbul, Turkey. [117]

Application of non-Newtonian calculus to function spaces was made by Ahmet Faruk Cakmak (Yildiz Technical University in Turkey) and Feyzi Basar (Fatih University in Turkey) in their lecture at the 2012 conference The Algerian-Turkish International Days on Mathematics, at University of Badji Mokhtar at Annaba, in Algeria. [127]

Application of non-Newtonian calculus to "continuous and bounded functions over the field of non-Newtonian complex numbers" was made by Zafer Cakir (Gumushane University, Turkey) in his lecture at the 2012 conference The Algerian-Turkish International Days on Mathematics, at University of Badji Mokhtar at Annaba, in Algeria. [137]

Non-Newtonian calculus is one of the topics of discussion at the 2013 Algerian-Turkish International Days on Mathematics conference at Fatih University in Istanbul, Turkey. [138]

Non-Newtonian calculus was used by Z. Avazzadeh, Z. Beygi Rizi, G. B. Loghmani, and F. M. Maalek Ghaini ((the first three from Yazd University in Iran and the last from Islamic Azad University in Iran) to devise a numerical method for solving nonlinear Volterra integro-partial differential equations. [124]

Weighted non-Newtonian calculus was used by ZHENG Xu and LI Jian-Zhong (School of Computer Science and Technology, Harbin Institute of Technology, Harbin, China) in their work on wireless sensor networks in computer science. [125]

The geometric integral is useful in stochastics. [22]

Non-Newtonian calculus has application to growth processes. [34, 82, 90, 104, 106]

Non-Newtonian calculus and some of its applications are the topics of an article by Ali Ozyapici at Eastern Mediterranean University in Cyprus in 2005. [113]

The geometric calculus and the bigeometric calculus were used by Hatice Aktore (Eastern Mediterranean University in Cyprus) in an article on multiplicative Runge-Kutta Methods. [133]

The geometric calculus was used in statistics and data analysis by Jarno van Roosmalen (Eindhoven University of Technology in the Netherlands) in his bachelor project on "multiplicative principal component analysis". [120]

Non-Newtonian calculus may have application in situations involving discontinuous phenomena. [35]

Application of geometric arithmetic ([15], [11], [10]) to multiplicative metric-spaces and multiplicative contraction-mappings was made by Muttalip Ozavsar and Adem C. Cevikel (both of Yildiz Technical University in Turkey). [128]

The geometric calculus was the subject of Christopher Olah's lecture at the Singularity Summit on 13 October 2012. [134] Singularity University's Singularity Summit is a conference on robotics, artificial intelligence, brain-computer interfacing, and other emerging technologies including genomics and regenerative medicine. [135] Christopher Olah is a Thiel Fellow.

NOTE. It is natural to speculate about future applications of non-Newtonian calculus, and of related matters such as weighted calculus, meta-calculus, averages, and means. Perhaps scientists, engineers, and mathematicians will use them to define new concepts, to yield new or simpler laws, or to formulate or solve problems.

0.5 Citations

"Non-Newtonian Calculus" [15] is cited in the book "The Rainbow of Mathematics: A History of the Mathematical Sciences" by the eminent mathematics-historian Ivor Grattan-Guinness. [6]

"Non-Newtonian Calculus" is cited in Gordon Mackay's book "Comparative Metamathematics". (The eighteen previous editions of "Comparative Metamathematics" are entitled "The True Nature of Mathematics".) [139]

Non-Newtonian calculus is cited in an article on atmospheric temperature by Robert G. Hohlfeld, Thomas W. Drueding, and John F. Ebersole. [89]

"Non-Newtonian Calculus" [15] is cited in an article on means by Jane Tang. [20]

"The First Systems of Weighted Differential and Integral Calculus" [9] is cited by the authors indicated below in their article on the global burden of cholera: Mohammad Ali, Anna Lena Lopez, Young Ae You, Young Eun Kim, Binod Sah, Brian Maskery, and John Clemens (all of the United Nations' International Vaccine Institute, Snu Research Park, San 4-8 Nakseongdae-dong Gwanak-gu, Seoul, Korea, 151 - 919). [98]

"The First Systems of Weighted Differential and Integral Calculus" [9] is cited by the authors indicated below in their article on thermochemistry of ammonium based ionic liquids: Sergey P. Verevkin and Vladimir N. Emer'yanenko (both of the University of Rostock, in Germany), Ingo Krossing (University of Freiburg, in Germany), and Roland Kalb (Proionic Production of Ionic Substances GmbH, in Graz, Austria). [108]

The geometric calculus is cited in a book on the phenomena of growth and structure-building by Manfred Peschel and Werner Mende (both of the German Academy of Sciences Berlin). [25]

The geometric calculus is the topic of an article by Dick Stanley of the University of California at Berkeley. [19]

The geometric calculus is the topic of an article and a seminar by Michael Coco of Lynchburg University. [115]

Non-Newtonian calculus is cited in a book on the energy crisis by R. Gagliardi and Jerry Pournelle. [26]

"Non-Newtonian Calculus" is cited in a doctoral thesis on nonlinear dynamical systems by David Malkin at University College London. [36]

The geometric calculus is cited by Daniel Karrasch in his doctoral thesis "Hyperbolicity and invariant manifolds for finite time processes" at the Technical University of Dresden in Germany. [141]

Non-Newtonian calculus is cited in the eBook "Economic Statistics". [91]

Non-Newtonian differentiation was the topic of a lecture by Karol Kosar and Ivan Kupka at a student conference at Comenius University in Slovakia. [93]

"The First Nonlinear System of Differential And Integral Calculus" [11] is cited in the article "L'Hopital's rule and Taylor's Theorem for product calculus" by Alex Twist and Michael Spivey. [103]

"Non-Newtonian Calculus" is cited in an article on petroleum engineering by Raymond W. K. Tang and William E. Brigham (both of Stanford University). [37]

"The First Nonlinear System of Differential and Integral Calculus" was cited in a lecture presented by Bruno Curko at the 2011 annual international symposium "Days of Frane Petric - From Petric to Boskovic" at Cres, Croatia. [109]

Non-Newtonian calculus is cited in a book on popular-culture by Paul Dickson. [28]

"Non-Newtonian Calculus" is cited in the journal Search. [77]

"Non-Newtonian Calculus" is cited in the journal Science Weekly. [64]

"Non-Newtonian Calculus" is cited in the journal Annals of Science. [66]

"Non-Newtonian Calculus" is cited in the journal Science Progress. [67]

"Non-Newtonian Calculus" is cited in the journal Allgemeines Statistisches Archiv. [69]

"Non-Newtonian Calculus" is cited in the journal Il Nuovo Cimento della Societa Italiana di Fisica: A. [70]

"The First Systems of Weighted Differential and Integral Calculus" [9] is cited in the journal Praxis der Mathematik. [79]

"Meta-Calculus: Differential and Integral" [7] is cited in the journal Indian Journal of theoretical physics. [80]

Each of the following two books is cited in the journal Publicationes Mathematicae. [56] 1) "Non-Newtonian Calculus": Volume 19, page 351, 1972. 2) "Bigeometric Calculus: A System with a Scale-Free Derivative": Volume 32, page 282, 1985.

Each of the following two books is cited in the journal Acta Scientiarum Mathematicarum. [60] 1) "Non-Newtonian Calculus": Volume 33, page 361, 1972. 2) "The First Nonlinear System of Differential and Integral Calculus": Volumes 42-43, page 225, 1980.

Each of the following six books is cited in the journal Industrial Mathematics. [61] 1) "Non-Newtonian Calculus": Volumes 43-45, page 91, 1994 . 2) "The First Nonlinear System of Differential and Integral Calculus": Volumes 28-30, page 143, 1978. 3) "The First Systems of Weighted Differential and Integral Calculus": Volumes 31-33, page 66, 1981. 4) "Meta-Calculus: Differential and Integral": Volumes 31-33, page 83, 1981. 5) "Bigeometric Calculus: A System with a Scale-Free Derivative": Volumes 33-34, page 91, 1983. 6) "Averages: A New Approach": Volumes 33-34, page 91, 1983.

Each of the following two books is cited in the journal Economic Books: Current Selections. [81] 1) "The First Systems of Weighted Differential and

Integral Calculus": Volume 9, page 29, 1982. 2) "Meta-Calculus: Differential and Integral": Volume 9, page 29, 1982.

Non-Newtonian calculus is cited in the journal Science Education International. [38]

Non-Newtonian calculus is cited in the journal Ciencia e cultura. [39]

Non-Newtonian calculus is cited in the journal American Statistical Association: 1997 Proceedings of the section on Bayesian Statistical Science. [40]

"Non-Newtonian Calculus" is cited in the Australian Journal of Statistics. [73]

"Non-Newtonian Calculus" is cited in the journal Physique au Canada. [83]

"Non-Newtonian Calculus" is cited in the journal Synthese. [74]

"Non-Newtonian Calculus" is cited in the journal Mathematical Education. [75]

"Non-Newtonian Calculus" is cited in the the journal Institute of Mathematical Statistics Bulletin. [76]

0.6 Reviews

Non-Newtonian Calculus was reviewed by Ivor Grattan-Guinness in the journal Middlesex Math Note . [101] Excerpt: "There is enough here [in Non-Newtonian Calculus] to indicate that non-Newtonian calculi ... have considerable potential as alternative approaches to traditional problems. This very original piece of mathematics will surely expose a number of missed opportunities in the history of the subject."

Each of the following six books was reviewed in the journal Internationale Mathematische Nachrichten. [53] 1) Non-Newtonian Calculus [15]: Number 105, 1972. Excerpt: "The possibilities opened up by the [non-Newtonian] calculi seem to be immense." 2) The First Nonlinear System of Differential and Integral Calculus [11]: Volumes 35-36, page 42, 1981. 3) The First Systems of Weighted Differential and Integral Calculus [9]: Volumes 35-36, page 40, 1981. 4) Meta-Calculus: Differential and Integral [7]: Volumes 35-36, page 140, 1981. 5) Bigeometric Calculus: A System with a Scale-Free Derivative [10]: Volumes 37-38, page 266, 1983. 6) Averages: A New Approach [8]: Volumes 37-38, page 266, 1983. Non-Newtonian Calculus was reviewed by David Pearce MacAdam in the Journal of the Optical Society of America. [100] Excerpt: "This [Non-Newtonian Calculus] is an exciting little book. ... The greatest value of these non-Newtonian calculi may prove to be their ability to yield simpler physical laws than the Newtonian calculus. Throughout, this book exhibits a clarity of vision characteristic of important mathematical creations. ... The authors have written this book for engineers and scientists, as well as for mathematicians. ... The writing is clear, concise, and very readable. No more than a working knowledge of [classical] calculus is assumed."

Each of the following five books was reviewed by Ralph P. Boas, Jr. in the journal Mathematical Reviews. [47] 1) The First Nonlinear System of Differential and Integral Calculus [11]: Mathematical Reviews, 1980. 2) The First Systems of Weighted Differential and Integral Calculus [9]: Mathematical Reviews,

1981. 3) Meta-Calculus: Differential and Integral [7]: Mathematical Reviews, 1982. 4) Bigeometric Calculus: A System with a Scale-Free Derivative [10]: Mathematical Reviews, 1984. Excerpt: "It seems plausible that people who need to study functions from this point of view might well be able to formulate problems more clearly by using [bigeometric] calculus instead of [classical] calculus." 5) Averages: A New Approach [8]: Mathematical Reviews, 1984.

Non-Newtonian Calculus [15] was reviewed in the journal Mathematical Reviews in 1978. [47]

Non-Newtonian Calculus was reviewed in the magazine Choice. [41]

Non-Newtonian Calculus was reviewed in the journal American Mathematical Monthly. [48]

The First Nonlinear System of Differential And Integral Calculus [11], a book about the geometric calculus, was reviewed in the journal American Mathematical Monthly. [52]

Bigeometric Calculus: A System with a Scale-Free Derivative [10] was reviewed in Mathematics Magazine. [49]

Bigeometric Calculus: A System with a Scale-Free Derivative was reviewed in the journal The Mathematics Student. [58]

The article "An introduction to non-Newtonian calculus" [12] was reviewed by K. Strubecker in the journal Zentralblatt Math (Zbl 0418.26008) [43].

The article "A new approach to means of two positive numbers" [14] was reviewed in Zentralblatt Math (Zbl 0586.26014) [43].

Each of the following three books was reviewed by K. Strubecker in Zentralblatt MATH [43]. 1) Non-Newtonian Calculus [15]: Zbl 0228.26002. 2) The First Systems of Weighted Differential and Integral Calculus [9]: Zbl 0443.26005. 3) Meta-Calculus: Differential and Integral [7]: Zbl 0493.26001.

The article "A new approach to means of two positive numbers" [14] was reviewed in the journal ZDM (1986c.10787) [50].

Each of the following five books was reviewed in ZDM [50]. 1) Non-Newtonian Calculus[15]: 1982a.00259. 2) The First Nonlinear System of Differential and Integral Calculus [11]: 1982a.00243. 3) The First Systems of Weighted Differential and Integral Calculus [9]: 1982a.00248. 4) Bigeometric Calculus: A System with a Scale-Free Derivative [10]: 19861.06868. 5) Averages: A New Approach [8]: 19861.06873.

0.7 Comments

"Your ideas [in "Non-Newtonian Calculus"] seem quite ingenious." - Professor Dirk J. Struik, Massachusetts Institute of Technology, USA; from his correspondence with Grossman, Grossman, and Katz.

"The monographs on non-Newtonian calculus by [Grossman, Grossman, and Katz] appear to be very useful and innovative." - Professor Kenneth J. Arrow, Nobel-Laureate, Stanford University, USA ; from his correspondence with Grossman, Grossman, and Katz.

"... Grossman and Katz ("Non-Newtonian Calculus", Lee Press, Pigeon Cove, Massachusetts, 1972) introduced the non-Newtonian calculi, [including] the branches of geometric, anageometric, and bigeometric calculi." - Professor Ahmet Faruk Cakmak, Yildiz Technical University, Turkey/ Professor Feyzi Basar, Fatih University, Turkey; from their article [122].

"In [1967] Michael Grossman and Robert Katz gave definitions of a new kind of derivative and integral ... and thus established a new calculus, called multiplicative calculus [the geometric calculus]. ... We think that [the geometric calculus] can especially be useful as a mathematical tool for economics and finance ... In the present paper our aim is to bring [the geometric] calculus to the attention of researchers ... and [to] demonstrate its usefulness." - Professor Agamirza E. Bashirov, Eastern Mediterranean University, Cyprus/ Professor Emine Misirli Kurpinar , Ege University, Turkey/ Professor Ali Ozyapici, Ege University, Turkey; from their article [2].

"In this study it becomes evident that the [geometric] calculus methodology has some advantages over [classical] calculus in modeling some processes in areas such as actuarial science, finance, economics, biology, demographics, etc." - Professor Agamirza E. Bashirov, Eastern Mediterranean University, Cyprus/ Professor Emine Misirli, Ege University, Turkey/ Professor Yucel Tandogdu, Eastern Mediterranean University, Turkey/ Professor Ali Ozyapici, Lefke European University, Turkey; from their article [94].

"We advocate the use of an alternative calculus in biomedical image analysis, known as multiplicative (a.k.a. non-Newtonian) calculus. ... The purpose of this article is to provide a condensed review of multiplicative calculus and to illustrate its potential use in biomedical image analysis." (The "multiplicative calculus" referred to here is the geometric calculus.) - Professors Luc Florack and Hans van Assen, both of Eindhoven University of Technology in The Netherlands; from their article [88].

"In this paper, we have tried to present how a non-Newtonian calculus could be applied to repostulate and analyse the neoclassical [Solow-Swan] exogenous growth model [in economics]. ... In fact, one must acknowledge that it is only under the effort of Grossman and Katz (1972) ... that such a non-Newtonian calculus emerged to give a natural answer to many growth phenomena. ... We must underscore that to discover that there was a non-Newtonian way to look to differential equations has been a great surprise for us. It opens the question to know if there are major fields of economic analysis which can be profoundly rethought in the light of this discovery." - Professor Diana Andrada Filip, Babes-Bolyai University of Cluj-Napoca, Romania/ Professor Cyrille Piatecki, Orleans University, France; from their article [82].

""Non-Newtonian Calculus", by Michael Grossman and Robert Katz, is a fascinating and (potentially) extremely important piece of mathematical theory. That a whole family of differential and integral calculi, parallel to but nonlinear with respect to ordinary Newtonian (or Leibnizian) calculus, should have remained undiscovered (or uninvented) for so long is astonishing - but true. Every mathematician and worker with mathematics owes it to himself to look into the discoveries of Grossman and Katz." - Professor James R. Meginniss, Claremont Graduate School and Harvey Mudd College, USA; from his correspondence with Grossman, Grossman, and Katz.

"There is enough here [in "Non-Newtonian Calculus"] to indicate that non-Newtonian calculi ... have considerable potential as alternative approaches to traditional problems. This very original piece of mathematics will surely expose a number of missed opportunities in the history of the subject." - Professor Ivor Grattan-Guinness, Middlesex University, England; from his review of "Non-Newtonian Calculus" [101].

"The possibilities opened up by the [non-Newtonian] calculi seem to be immense." - Professor H. Gollmann, Graz, Austria; from his review of "Non-Newtonian Calculus" [53].

"This ["Non-Newtonian Calculus"] is an exciting little book. ... The greatest value of these non-Newtonian calculi may prove to be their ability to yield simpler physical laws than the Newtonian calculus. Throughout, this book exhibits a clarity of vision characteristic of important mathematical creations. ... The authors have written this book for engineers and scientists, as well as for mathematicians. ... The writing is clear, concise, and very readable. No more than a working knowledge of [classical] calculus is assumed." - Professor David Pearce MacAdam, Cape Cod Community College, USA; from his review of "Non-Newtonian Calculus" [100].

"In our proseminar we'll learn some of the most exciting non-linear calculations and consider the applications for which they may be of particular interest. The applications range from rates of return and other growth processes to highly active areas of digital image processing." - Professor Joachim Weickert, Saarland University, Germany; from his description of his course on non-Newtonian calculus at Saarland University in 2012. [106]

"Many natural phenomena, from microscopic bacteria growth, through macroscopic turbulence, to the large scale structure of the Universe, display a fractal character. For studying the time evolution of such "rough" objects, the classical, "smooth" Newtonian calculus is not enough." - Professor Wojbor Woycznski, Case Western Reserve University, USA; from an abstract to his seminar on "non-Newtonian calculus for the dynamics of random fractal structures". [104]

"While one problem can be easily expressed in one calculus, the same problem can not be expressed as easily [in another]." - Professors Emine Misirli and Yusuf Gurefe, both of Ege University in Turkey; from their article [123].

"It seems plausible that people who need to study functions from this point of view might well be able to formulate problems more clearly by using [bigeometric] calculus instead of [classical] calculus." - Professor Ralph P. Boas, Jr., Northwestern University, USA; from his review of "Bigeometric Calculus: A System with a Scale-Free Derivative" [47].

"The more innovative parts of the Stern Review - the non-Newtonian calculus in Chapter 13, for instance - ..." - Professor Richard Tol, University of Sussex, England; from his forward to "What is Wrong with Stern?", a critique of "Stern Review on the Economics of Climate Change". [116]

"This work presents a new operator of non-Newtonian type which [has] shown [to] be more efficient in contour detection [in images with multiplicative noise] than the traditional operators. ... In our view, the work proposed in (Grossman and Katz, 1972) stands as a foundation ... Innovative applications of non-Newtonian calculus can be found in the field of Bayesian Analysis (Meginniss, 1980)." [15, 16] - Professors Marco Mora, Fernando Cordova-Lepe, and Rodrigo Del-Valle, all of Universidad Catolica del Maule in Chile; from their article [99].

0.8 Quotations

"For each successive class of phenomena, a new calculus or a new geometry, as the case might be, which might prove not wholly inadequate to the subtlety of nature." - Henry John Stephen Smith, as quoted in Nature, Volume 8 (1873), page 450.

"In general the position as regards all such new calculi is this - That one cannot accomplish by them anything that could not be accomplished without them. However, the advantage is, that, provided such a calculus corresponds to the inmost nature of frequent needs, anyone who masters it thoroughly is able - without the unconscious inspiration of genius which no one can command - to solve the respective problems, indeed to solve them mechanically in complicated cases in which, without such aid, even genius becomes powerless. Such is the case with the invention of general algebra, with the differential calculus, and in a more limited region with Lagrange's calculus of variations, with my calculus of congruences, and with Mobius's calculus. Such conceptions unite, as it were, into an organic whole countless problems which otherwise would remain isolated and require for their separate solution more or less application of inventive genius." - Carl Friedrich Gauss, as quoted in "Gauss, Werke, Bd. 8", page 298; and as quoted in "Memorabilia Mathematica" (or "The Philomath's Quotation Book") (1914) by Robert Edouard Moritz, quotation 1215.

"A large part of mathematics which becomes useful developed with absolutely no desire to be useful, and in a situation where nobody could possibly know in what area it would become useful; and there were no general indications that it would ever be so. By and large it is uniformly true in mathematics that there is a time lapse between a mathematical discovery and the moment when it is useful; and that this lapse of time can be anything from 30 to 100 years, in some cases even more; and that the whole system seems to function without any direction, without any reference to usefulness, and without any desire to do things which are useful." - John von Neumann, as quoted in "Out of the Mouths of Mathematicians: A Quotation Book for Philomaths" (1993) by R. Schmalz.

"But if some mind very different from ours were to look upon some property of some curved line as we do on the evenness of a straight line, he would not recognize as such the evenness of a straight line; nor would he arrange the elements of his geometry according to that very different system, and would investigate quite other relationships as I have suggested in my notes. "We fashion our geometry on the properties of a straight line because that seems to us to be the simplest of all. But really all lines that are continuous and of a uniform nature are just as simple as one another. Another kind of mind which might form an equally clear mental perception of some property of any one of these curves, as we do of the congruence of a straight line, might believe these curves to be the simplest of all, and from that property of these curves build up the elements of a very different geometry, referring all other curves to that one, just as we compare them to a straight line. Indeed, these minds, if they noticed and formed an extremely clear perception of some property of, say, the parabola, would not seek, as our geometers do, to rectify the parabola, they would endeavor, if one may coin the expression, to parabolify the straight line." - Roger Joseph Boscovich, as quoted in "Boscovich's mathematics", an article by J. F. Scott, in the book "Roger Joseph Boscovich" (1961) edited by Lancelot Law Whyte; and as quoted in "Transient pressure analysis in composite reservoirs" (1982) by Raymond W. K. Tang and William E. Brigham.

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