product of divisors function*

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The product of all positive divisors of a nonzero integer n is equal $\sqrt{n^{\tau(n)}}$, where tau function $\tau(n)$ expresses the number of the positive divisors of n.

Proof. Let $t = \tau(n)$ and the positive divisors of n be $a_1 < a_2 < \ldots < a_t$.

If n is not a square of an integer, t is even (see parity of τ function), whence

$$\begin{cases} a_1 a_t &= n \\ a_2 a_{t-1} &= n \\ \quad \cdots & \\ a_{\frac{t}{2}} a_{\frac{t+2}{2}} &= n. \end{cases}$$

Thus

$$\prod_{d|n} d \;=\; a_1 a_2 \cdots a_t \;=\; n^{\frac{t}{2}}.$$

If n is a square of an integer, t is odd, and we have

$$\begin{cases} a_1 a_t &= n \\ a_2 a_{t-1} &= n \\ \quad \cdots & \\ a_{\frac{t-1}{2}} a_{\frac{t+3}{2}} &= n \\ a_{\frac{t+1}{2}} &= n^{\frac{1}{2}}. \end{cases}$$

In this case we obtain a similar result:

$$\prod_{d|n} d \;=\; a_1 a_2 \cdots a_t \;=\; n^{\frac{t-1}{2}+\frac{1}{2}} \;=\; n^{\frac{t}{2}}$$

Note. The absolute value of the product of all divisors is $n^{\tau(n)}$.

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