morphic number*

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The golden ratio $\varphi = \frac{1+\sqrt{5}}{2}$ satisfies the equations

$$\begin{cases} \varphi+1 \;=\; \varphi^2, \\ \varphi-1 \;=\; \varphi^{-1} \end{cases} \tag{1}$$

from which the latter is obained from the former by dividing by φ. There is a similar pair of equations satisfied by the plastic number P:

$$\begin{cases} P+1 \;=\; P^3, \\ P-1 \;=\; P^{-4} \end{cases} \tag{2}$$

Here, the latter equation is justified by

$$P^5-P^4-1 \;\equiv\; (\underbrace{P^3-P-1}_{=\,0})(P^2-P+1)$$

when this is divided by P^4.

An algebraic integer is called a *morphic number*, iff it satisfies a pair of equations

$$\begin{cases} x+1 \;=\; x^m, \\ x-1 \;=\; x^{-n} \end{cases} \tag{3}$$

for some positive integers m and n.

Accordingly, the golden ratio and the plastic number are morphic numbers. It can be shown that there are no other real morphic numbers greater than 1.

References

[1] J. Aarts, R. Fokkink, G. Kruijtzer: Morphic numbers. – *Nieuw Archief voor Wiskunde* 5/2 (2001).

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