

well-definedness of product of finitely generated ideals*

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Let R be of a commutative ring with nonzero unity. If

$$\mathfrak{a} = (a_1, \dots, a_m) = (\alpha_1, \dots, \alpha_\mu) \quad (1)$$

and

$$\mathfrak{b} = (b_1, \dots, b_n) = (\beta_1, \dots, \beta_\nu) \quad (2)$$

are two finitely generated ideals of R , both with two generating systems, then the ideals

$$\mathfrak{c} := (a_1b_1, \dots, a_ib_j, \dots, a_mb_n)$$

and

$$\mathfrak{d} := (\alpha_1\beta_1, \dots, \alpha_i\beta_j, \dots, \alpha_\mu\beta_\nu)$$

are equal.

Proof. By (1) and (2), for every i, j , there are elements r_{ik}, s_{jl} of R such that

$$a_i = r_{i1}\alpha_1 + \dots + r_{i\mu}\alpha_\mu, \quad b_j = s_{j1}\beta_1 + \dots + s_{j\nu}\beta_\nu. \quad (3)$$

Multiplying the equations (3) we see that

$$a_ib_j = (r_{i1}s_{j1})(\alpha_1\beta_1) + (r_{i2}s_{j1})(\alpha_2\beta_1) + \dots + (r_{i\mu}s_{j\nu})(\alpha_\mu\beta_\nu),$$

whence the generators a_ib_j of \mathfrak{c} belong to \mathfrak{d} and consequently $\mathfrak{c} \subseteq \mathfrak{d}$. The reverse containment is seen similarly.

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