A CATEGORY THEORY AND HIGHER DIMENSIONAL ALGEBRA APPROACH TO COMPLEX SYSTEMS BIOLOGY, META-SYSTEMS AND ONTOLOGICAL THEORY OF LEVELS:*

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A CATEGORY THEORY AND HIGHER DIMENSIONAL ALGEBRA APPROACH TO COMPLEX SYSTEMS BIOLOGY,

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META-SYSTEMS AND ONTOLOGICAL THEORY OF LEVELS: *EMERGENCE OF LIFE, SOCIETY, HUMAN CONSCIOUSNESS AND ARTIFICIAL INTELLIGENCE*

I. C. Baianu, James F. Glazebrook and Ronald Brown

Abstract.

An attempt is made from the viewpoint of the recent theory of ontological levels [2],[40],[137],[206]-[209] to understand the origins and emergence of life, the dynamics of the evolution of organisms and species, the ascent of man and the co-emergence, as well as co-evolution of human consciousness within organised societies. The new concepts developed for understanding the emergence and evolution of life, as well as human consciousness, are in terms of globalisation of multiple, underlying processes into the meta-levels of their existence. Such concepts are also useful in computer aided ontology and computer science [1],[194],[197]. In this monograph we present a novel approach to the problems raised by higher complexity in both nature and the human society, by considering the highest and most complex levels of objective existence as ontological meta-levels, such as those present in the creative human minds and civilised, modern societies. Thus, a collection of sets may be a *class*, instead of a set [59],[176]-[177]; it may also be called a 'super-set', or a *meta-set*; a 'theorem' about theorems is a *meta-theorem*, and a 'theory' about theories is a *'meta-theory'*. In the same sense that a statement about propositions is a higher-level ⟨*proposition*⟩ rather than a simple proposition, a global process of subprocesses is a *meta-process*, and the emergence of higher levels of reality *via* such meta-processes results in the objective existence of *ontological meta-levels*. It is also attempted here to classify more precisely the levels of reality and species of organisms than it has been thus far reported. The selected approach for our broad– but in-depth– study of the fundamental, relational structures and functions present in living, higher organisms and of the extremely complex processes and meta-processes of the human mind combines new concepts from three recently developed, related mathematical fields: Algebraic Topology, Category Theory (CT) and Higher Dimensional Algebra (HDA). Several important relational structures present in organisms and the human mind are naturally represented in terms of universal CT concepts, variable topology, non-Abelian categories and HDA-based notions. Such relatively new concepts are defined in the appropriate sequence beginning with the concept of groupoid which is fundamental to all algebraic topology studies [63], [69], and that also turns out to be essential to numerous applications in mathematical biology [11]-[23],[34],[74], including those of higher dimensional groupoids in theoretical neuroscience [38],[69]-[70].

An unifying theme of local-to-global approaches to organismal development, biological evolution and human consciousness leads to novel patterns of relations that emerge in super- and ultra- complex systems in terms of global compositions of local procedures [33],[39]. This novel algebraic topology concept of *combination of local procedures* is suggested to be relevant to both ontogenetic development and organismal evolution, beginning with the origin of species of higher organisms. Fundamentally inter–related, higher homotopy and holonomy groupoid concepts may provide a formal framework for an improved understanding of evolutionary biology and the origin of species on multiple levels–from molecular to species and biosphere levels. All key concepts pertaining to this context are here defined for a self-contained presentation, notwithstanding the difficulties associated with understanding the essence of life, the human mind, consciousness and its origins. One can define pragmatically the human brain in terms of its neurophysiological functions, anatomical and microscopic structure, but one cannot as readily observe and define the much more elusive human mind which depends both upon a fully functional human brain and its training or education by the human society. Human minds that do not but weakly interact with those of any other member of society are partially disfunctional, and this creates increasing problems with the society integration of large groups of people that only interact weakly with all the other members of society. Obviously, it does take a fully functional mind to observe and understand the human mind. It is then claimed that human consciousness is an *unique* phenomenon which should be regarded as a composition, or combination of ultra-complex, global processes of subprocesses, at a *meta-level* not sub–summed by, but compatible with, human brain dynamics [11]–[23],[33]. Thus, a defining characteristic of such conscious processes involves a *combination of global procedures* or meta-processes– such as the parallel processing of both image and sound sensations, perceptions and emotions, decision making and learned reflexes, etc.– that ultimately leads to the ontological meta-level of the ultra-complex, human mind. In this monograph we shall not attempt to debate if other species are capable of consciousness, or to what extent, but focus instead on the ultra-complex problems raised by human consciousness and its emergence. Current thinking [87], [91],[182],[186],[188], [190], [195]-[196],[203],[247] considers the actual emergence of human consciousness [83],[91],[186],[190],[261] –and also its ontic category– to be critically dependent upon the existence of both a human society level of *minimal* (tribal) organization [91],[186],[190], and that of an extremely complex structural –functional unit –the human brain with an *asymmetric* network topology and a dynamic network connectivity of very high-order [187],[218], [262]. Then, an extension of the concept of coevolution of human consciousness and society leads one to the concept of *social consciousness* [190]. One arrives also at the conclusion that the human mind and consciousness are the result not only of the *co-evolution* of man and his society [91],[186],[190], but that they are, in fact, the result of the original *co-emergence* of the meta-level of a minimally-organized human society with that of several, ultra-complex human brains. Unlike the myth of only one Adam and one Eve being the required generator of

human society, our co-emergence concept leads necessarily to the requirement of several such 'primitive' human couples co-existing in order to generate both a minimally organized society and several, minimally self-conscious, interacting *H. sapiens* minds that shaped the first Rosetta groupoids of *H. sapiens* into human tribes. The human 'spirit' and society are, thus, *completely inseparable*–just like the very rare Siamese twins. Therefore, the appearance of human consciousness is considered to be critically dependent upon the societal co-evolution, the emergence of an elaborate language-symbolic communication system, as well as the existence of 'virtual', higher dimensional, non–commutative processes that involve separate space and time perceptions in the human mind. Two fundamental, logic adjointness theorems are considered that provide a logical basis for categorical representations of functional genome and organismal networks in variable categories and extended toposes, or topoi, 'classified' (or encoded) by multi-valued logic algebras; their subtly nuanced connections to the variable topology and multiple geometric structures of developing organisms are also pointed out. Theories of the mind are thus considered in the context of a novel ontological theory of levels. Our ultra-complexity viewpoint throws new light on previous semantic models in cognitive science and on the theory of levels formulated within the framework of Categorical Ontology [40],[69]. Our novel approach to meta-systems and levels using Category Theory and HDA mathematical representations is also applicable–albeit in a modified form–to supercomputers, complex quantum computers, man–made neural networks and novel designs of advanced artificial intelligence (AI) systems (AAIS). Anticipatory systems and complex causality at the top levels of reality are also discussed in the context of Complex Systems Biology (CSB), psychology, sociology and ecology. A paradigm shift towards *non-commutative*, or more generally, non-Abelian theories of highly complex dynamics [33],[40],[69] is suggested to unfold now in physics, mathematics, life and cognitive sciences, thus leading to the realizations of higher dimensional algebras in neurosciences and psychology, as well as in human genomics, bioinformatics and interactomics. The presence of strange attractors in modern society dynamics, and especially the emergence of new meta-levels of still-higher complexity in modern society, gives rise to very serious concerns for the future of mankind and the continued persistence of a multi-stable Biosphere if such ultra-complexity, meta-level issues continue to be ignored.

KEYWORDS: *Categorical Ontology of Super-Complex and Ultra-Complex System Dynamics,Higher Dimensional Algebra of Networks,Theoretical Biology and Variable Groupoids, Non-Abelian Quantum Algebraic Topology and Quantum Double Groupoids, Higher Homotopy-General van Kampen theorems; autistic children, advanced artificial intelligence and biomimetics*

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I. C. Baianu, James F. Glazebrook and Ronald Brown: Category Theory & Emergence of Life, Society, Human Consciousness & AI

1. INTRODUCTION

Ontology has acquired over time several meanings, and it has also been approached in many different ways, but all of these are connected to the concepts of an *'objective existence'* and categories of items. A related, important function of Ontology is to *classify and/or categorize* items and essential aspects of reality [2],[206]-[210]. We shall employ therefore the adjective *"ontological"* with the meaning of pertaining to objective, real existence in its essential aspects. We shall also consider here the noun *existence* as a basic, or primary concept which cannot be defined in either simpler or atomic terms, with the latter in the sense of Wittgenstein. Furthermore, generating *meaningful classifications of items* that belong to the objective reality is also a related, major task of ontology. Mathematicians specialised in Group Theory are also familiar with the classification problem into various types of the mathematical objects called groups. Computer scientists that carry out ontological classifications, or study AI and Cognitive Science [201], are also interested in the logical foundations of computer science [1],[194],[197],[201].

For us the most interesting question by far is how human consciousness and civilisation emerged subsequent only to the emergence of *H. sapiens.* This may have arisen through the development of speech-syntactic language and an appropriately organized 'primitive' society [91],[186] (perhaps initially made of hominins/hominides). No doubt, the details of this highly complex, emergence process have been the subject of intense controversies over the last several centuries, and many differing opinions, even among these authors, and they will continue to elude us since much of the essential data must remains either scarce or unattainable. It is however known that the use of cooked food, and so of fire, was necessary for the particular physiognomy of even *H. erectus*, as against other primates, and such use perhaps required a societal context several millenia even before this hominin, partly in terms of the construction of hearths, which were a necessity for the efficient cooking of food.

Other factors such as the better use of purposefully designed tools, simple weapons and the intense struggle for the survival of the fittest have also contributed greatly to the selective advantages of *H. sapiens* in the fierce struggle for its existence; nevertheless, there is an overwhelming consensus in the specialised literature that the *co-evolution* of the human mind and society was the predominant, or key factor for the survival of *H. sapiens* over that of all other closely related species in the genus *Homo* that did not survive– in spite of having existed earlier, and some probably much longer than *H. sapiens.*

The authors aim at a concise presentation of novel methodologies for studying such difficult, as well as controversial, ontological problems of Space and Time at different levels of objective reality defined here as Complex, Super–Complex and Ultra–Complex Dynamic Systems, simply in order 'to divide and conquer'. The latter two are biological organisms, human (and perhaps also hominide) societies, and more generally, variable 'systems' and meta-systems that are not recursively–computable. Rigorous definitions of the logical and mathematical concepts employed here, as well as a step-by-step construction of our conceptual framework, were provided in a recent series of publications on categorical ontology of levels and complex systems dynamics [33]-[34],[39]-[40]. The continuation of the very existence of human society may now depend upon an improved understanding of highly complex systems and the human mind, and also upon how the global human society interacts with the rest of the biosphere and its natural environment. It is most likely that such tools that we shall suggest here might have value not only to the sciences of complexity and Ontology but, more generally also, to all philosophers seriously interested in keeping on the rigorous side of the fence in their arguments. Following Kant's critique of 'pure' reason and Wittgenstein' s critique of language misuse in philosophy, one needs also to critically examine the possibility of using general and universal, mathematical language and tools in formal approaches to a rigorous, formal Ontology. Throughout this monograph we shall use the attribute *'categorial'* only for philosophical and linguistic arguments. On the other hand, we shall utilize the rigorous term *'categorical'* only in conjunction with applications of concepts and results from the more restrictive, but still quite general, mathematical *Theory of Categories, Functors and Natural Transformations* (TC-FNT). According to SEP (2006): "Category theory ... is a general mathematical **theory of structures and of systems of structures**. *Category theory is both an interesting object of philosophical study, and a potentially powerful formal tool for philosophical investigations of concepts such as space, system, and even truth... It has come to occupy a central position in contemporary mathematics and theoretical computer science, and is also applied to mathematical physics.*" [248]. Traditional, modern philosophy– considered as a search for improving knowledge and wisdom– does also aims at unity that might be obtained as suggested by Herbert Spencer in 1862 through a *'synthesis of syntheses'*; this could be perhaps iterated many times because each treatment is based upon a critical evaluation and provisional improvements of previous treatments or stages. One notes however that this methodological question is hotly debated by modern philosophers beginning, for example, by Descartes before Kant and Spencer; Descartes championed with a great deal of success the *'analytical'* approach in which *all* available evidence is, in principle, examined critically and skeptically first both by the proposer of novel metaphysical claims and his, or her, readers. Descartes equated the 'synthetic' approach with the Euclidean 'geometric' (axiomatic) approach, and thus relegated synthesis to a secondary, perhaps less significant, role than that of critical *analysis* of scientific 'data' input, such as the laws, principles, axioms and theories of all specific sciences. Spinoza's, Kant's and Spencer's styles might be considered to be synthetic by Descartes and all Cartesians, whereas Russell's approach might also be considered to be analytical. Clearly and correctly, however, Descartes did not regard analysis (A) and synthesis (S) as exactly inverse to each other, such as AS, and also not merely as 'bottom–up' and 'top–bottom' processes (↓↑). Interestingly, unlike Descartes' discourse of the philosophical method, his treatise of philosophical principles comes closer to the synthetic approach in having definitions and deductive attempts, logical inferences, not unlike his 'synthetic' predecessors, albeit with completely different claims and perhaps a wider horizon. The reader may immediately note that if one, as proposed by Descartes, begins the presentation or method with an analysis A, followed by a synthesis S, and then reversed the presentation in a follow-up treatment by beginning with a synthesis $S*$ followed by an analysis A' of the predictions made by S' consistent, or analogous, with A, then obviously $AS \neq S'A'$ because we assumed that $A \simeq A'$ and that $S \neq S'$. Furthermore, if one did not make any additional assumptions about analysis and synthesis, then $analysis \rightarrow synthesis \neq synthesis \rightarrow analysis$, or $AS \neq SA$, that is analysis and synthesis obviously *'do not commute'*; such a theory when expressed mathematically would be then called *'non-Abelian'*. This is also a good example of the meaning of the term non-Abelian in a philosophical, epistemological context.

2.The Theory of Levels in Categorial and Categorical Ontology

This section outlines our novel methodology and approach to the ontological theory of levels, which is then applied in subsequent sections in a manner consistent with our recently published developments [33]-[34],[39]-[40]. Here, we are in harmony with the theme and approach of Poli's ontological theory of levels of reality [121], [206]–[211]) by considering both philosophical–categorial aspects such as Kant's relational and modal categories, as well as categorical–mathematical tools and models of complex systems in terms of a dynamic, evolutionary viewpoint.

We are then presenting a Categorical Ontology of highly complex systems, discussing the modalities and possible operational logics of living organisms, in general. Then, we consider briefly those integrated functions of the human brain that support the ultra-complex human mind and its important roles in societies. Mores specifically, we propose to combine a critical analysis of language with precisely defined, abstract categorical concepts from Algebraic Topology reported by Brown et al, in 2007 [69], and the general-mathematical Theory of Categories, Functors and Natural Transformations: [56], [80], [98]-[102], [105]-[106],[113],[115-[119],[130], [133]-[135],[141]-[143], [151],[154], [161]-[163],[165]-[168], [172], [175]-[177],[183], [192]-[194],[198]-[199] [213]-[215],[225], [227],[246], [252], [256] into a categorical framework which is suitable for further ontological development, especially in the relational rather than modal ontology of complex spacetime structures. Basic concepts of Categorical Ontology are presented in this section, whereas formal definitions are relegated to one of our recent, detailed reports [69]. On the one hand, philosophical categories according to Kant are: *quantity, quality, relation* and *modality*, and the most complex and far-reaching questions concern the relational and modality-related categories. On the other hand, mathematical categories are considered at present as the most general and universal structures in mathematics, consisting of related *abstract objects connected by arrows.* The abstract objects in a category may, or may not, have a specified *structure*, but must all be of the same type or kind in any given category. The arrows (also called *'morphisms'*) can represent relations, mappings/functions, operators, transformations, homeomorphisms, and so on, thus allowing great flexibility in applications, including those outside mathematics as in: Logics [118]-[120], Computer Science [1], [161]-[163] [201],[248], [252], Life Sciences [5],[11]-[17],[19],[23],[28]-[36],[39],[40],[42]-[44],[70],[74],[103]-[104],[230],[232],[234]-[238],[264], Psychology, Sociology [33],[34],[39],[40],[74], and Environmental Sciences [169]. The mathematical category also has a form of *'internal' symmetry*, specified precisely as the *commutativity* of chains of morphism compositions that are uni-directional only, or as *naturality of diagrams* of morphisms; finally, any object A of an abstract category has an associated, unique, identity, 1_A, and therefore, one can replace all objects in abstract categories by the identity morphisms. When all arrows are *invertible*, the special category thus obtained is called a *'groupoid'*, and plays a fundamental role in the field of mathematics called Algebraic Topology.

The categorical viewpoint– as emphasized by William Lawvere, Charles Ehresmann and most mathematicians– is that the key concept and mathematical structure is that of *morphisms* that can be seen, for example, as abstract relations, mappings, functions, connections, interactions, transformations, and so on. Thus, one notes here how the philosophical category of *'relation'* is closely allied to the basic concept of morphism, or arrow, in an abstract category; the implicit tenet is that *arrows are what counts.* One can therefore express all essential properties, attributes, and structures by means of arrows that, in the most general case, can represent either philosophical 'relations' or modalities, the question then remaining if philosophical–categorial properties need be subjected to the categorical restriction of *commutativity.* As there is no *a priori* reason in either nature or 'pure' reasoning, including any form of Kantian 'transcendental logic', that either relational or modal categories should in general have any symmetry properties, one cannot impose onto philosophy, and especially in ontology, all the strictures of category theory, and especially commutativity. Interestingly, the same comment applies to Logics: only the simplest forms of Logics, the Boolean and intuitionistic, Heyting-Brouwer logic algebras are commutative, whereas the algebras of many-valued (MV) logics, such as Łukasiewicz logic are *non-commutative* (or *non-Abelian*).

3. Basic Structure of Categorical Ontology. The Theory of Levels: Emergence of Higher Levels, Meta–Levels and Their Sublevels

With the provisos specified above, our proposed methodology and approach employs concepts and mathematical techniques from Category Theory which afford describing the characteristics and binding of ontological levels besides their links with other theories. Whereas Hartmann in 1952 stratified levels in terms of the four frameworks: physical, 'organic'/biological, mental and spiritual [137], we restrict here mainly to the first three. The categorical techniques which we introduce provide a powerful means for describing levels in both a linear and interwoven fashion, thus leading to the necessary bill of fare: emergence, complexity and open non-equilibrium, or irreversible systems. Furthermore, any effective approach to Philosophical Ontology is concerned with *universal items* assembled in categories of objects and relations, involving, in general, transformations and/or processes. Thus, Categorical Ontology is fundamentally dependent upon both space and time considerations. Therefore, one needs to consider first a dynamic classification of systems into different levels of reality, beginning with the physical levels (including the fundamental quantum level) and continuing in an increasing order of complexity to the chemical–molecular levels, and then higher, towards the biological, psychological, societal and environmental levels. Indeed, it is the principal tenet in the theory of levels that : *"there is a two-way interaction between social and mental systems that impinges upon the material realm for which the latter is the bearer of both"* [209]. Therefore, any effective Categorical Ontology approach requires, or generates–in the constructive sense–a ‘***structure***’ or *pattern of linked items* rather than a discrete set of items. The evolution in our universe is thus seen to proceed from the level of ‘elementary’ quantum ‘wave–particles’, their interactions *via* quantized fields (photons, bosons, gluons, etc.), also including the quantum gravitation level, towards aggregates or categories of increasing complexity. In this sense, the classical macroscopic systems are defined as ‘simple’ dynamical systems that are *computable recursively* as numerical solutions of mathematical systems of either ordinary or partial differential equations. Underlying such mathematical systems is always the Boolean, or chrysippian, logic, namely, the logic of sets, Venn diagrams, digital computers and perhaps automatic reflex movements/motor actions of animals. The simple dynamical systems are always recursively computable (see for example, Suppes, 1995–2006 [253]-[254], and also [23]), and in a certain specific sense, both degenerate and *non-generic*, and consequently also they are *structurally unstable* to small perturbations; such systems are, in general, deterministic in the classical sense, although there are arguments about the possibility of chaos in quantum systems. The next higher order of systems is then exemplified by ‘systems with chaotic dynamics’ that are conventionally called ‘complex’ by physicists who study ‘chaotic’ dynamics/Chaos theories, computer scientists and modelers even though such physical, dynamical systems are still completely deterministic. It has been formally proven that such ‘systems with chaos’ are *recursively non-computable* (see for example, refs. [23] and [28] for a 2-page, rigorous mathematical proof and relevant references), and therefore they cannot be completely and correctly simulated by digital computers, even though some are often expressed mathematically in terms of iterated maps or algorithmic-style formulas. Higher level systems above the chaotic ones, that we shall call *‘Super–Complex, Biological systems’*, are the living organisms, followed at still higher levels by the *ultra-complex ‘systems’* of the human mind and human societies that will be discussed in the last sections. The evolution to the highest order of complexity- the ultra-complex, meta–‘system’ of processes of the human mind–may have become possible, and indeed accelerated, only through human societal interactions and effective, elaborate/rational and symbolic communication through speech (rather than screech –as in the case of chimpanzees, gorillas, baboons, etc).

4. FUNDAMENTAL CONCEPTS OF ALGEBRAIC TOPOLOGY WITH POTENTIAL APPLICATION TO ONTOLOGY LEVELS THEORY AND THE CLASSIFICATION OF SPACETIME STRUCTURES

We shall consider in this section the potential impact of novel Algebraic Topology concepts, methods and results on the problems of defining and classifying rigorously Quantum Spacetimes (QSS)[3], [36]-[38],[69], [78]-[79]. The 600-page project manuscript, *'Pursuing Stacks'* written by Alexander Grothendieck in 1983 was partly aimed at a *non-Abelian homological algebra*; it did not achieve this goal but has been very influential in the development of weak n-categories and other *higher categorical structures* that are relevant to QSS structures. With the advent of Quantum Groupoids–generalizing Quantum Groups, Quantum Algebra and Quantum Algebraic Topology, several fundamental concepts and new theorems of Algebraic Topology may also acquire an increased importance through their potential applications to current problems in theoretical and mathematical physics, such as those described in an available preprint [38], and also in several other recent publications [36]–[37], [69]. In such novel applications, both the internal and external groupoid symmetries [265] may too acquire new physical significance. Thus, if quantum theories were to reject the notion of a *continuum* model for spacetime, then it would also have to reject the notion of the real line and the notion of a path. How then is one to construct a homotopy theory? One possibility is to take the route signalled by Čech [82], and which later developed in the hands of Borsuk into 'Shape Theory' [86]. Thus a quite general space is studied indirectly by means of its approximation by open covers. Yet another possible approach is briefly outlined in the next section.

Several fundamental concepts of Algebraic Topology and Category Theory that are needed throughout this monograph will be introduced next so that we can reach an extremely wide range of applicability, especially to the higher complexity levels of reality. Full mathematical details are also available in a recent paper by Brown et al. [69] that focused on a mathematical–conceptual framework for a formal approach to Categorical Ontology and the Theory of Ontological Levels [206], [40].

Groupoids, Topological Groupoids, Groupoid Atlases and Locally Lie Groupoids

Recall that a *groupoid* G is a small category in which every morphism is an isomorphism.

Topological Groupoids

An especially interesting concept is that of a *topological groupoid* which is a groupoid internal to the category Top; further mathematical details are presented in the paper by Brown et al. in 2007 [69].

Groupoid Atlases

Motivation for the notion of a groupoid atlas comes from considering families of group actions, in the first instance on the same set. As a notable instance, a subgroup H of a group G gives rise to a group action of H on G whose orbits are the cosets of H in G. However a common situation is to have more than one subgroup of G, and then the various actions of these subgroups on G are related to the actions of the intersections of the subgroups. This situation is handled by the notion of *global action*, as defined in [41]. A key point in this construction is that the orbits of a group action then become the connected components of a groupoid. Also this enables relations with other uses of groupoids. The above account motivates the following. A *groupoid atlas* $\mathcal{A}$ on a set $X_{\mathcal{A}}$ consists of a family of 'local groupoids' $(\mathsf{G}_{\mathcal{A}})$ defined with respective object sets $(X_{\mathcal{A}})_{\alpha}$ taken to be subsets of $X_{\mathcal{A}}$. These local groupoids are indexed by a set $\Psi_{\mathcal{A}}$, again called the *coordinate system of* $\mathcal{A}$ which is equipped with a reflexive relation denoted by . This data is to satisfy several conditions reported in [41] by Bak et al. in 2006, and also discussed in [63] in the context of Categorical Ontology.

The van Kampen Theorem and Its Generalisations to Groupoids and Higher Homotopy

The van Kampen Theorem has an important and also anomalous rôle in algebraic topology. It allows computation of an important invariant for spaces built up out of simpler ones. It is anomalous because it deals with a non-Abelian invariant, and has not been seen as having higher dimensional analogues. However, Brown found in 1967 a generalisation of this theorem to groupoids [60], stated as follows. In this, $\pi_1(X, X_0)$ is the *fundamental groupoid* of X on a set X_0 of base points: so it consists of homotopy classes rel end points of paths in X joining points of $X_0 \cap X$. Such methods were extended successfully by R. Brown to *higher dimensions*. The potential applications of the Higher Homotopy van Kampen Theorem [37]-38] were already discussed in a previous paper [69] published by Brown, Glazebrook and Baianu in 2007.

5. LOCAL-TO-GLOBAL PROBLEMS IN SPACETIME STRUCTURES. SYMMETRY BREAKING, IRREVERSIBILITY AND THE EMERGENCE OF HIGHLY COMPLEX DYNAMICS

Spacetime Local Inhomogeneity, Discreteness and Broken Symmetries: From Local to Global Structures.

On summarizing in this section the evolution of the physical concepts of space and time, we are pointing out first how the views changed from homogeneity and continuity to *inhomogeneity and discreteness.* Then, we link this paradigm shift to a possible, novel solution in terms of local-to-global approaches and procedures to spacetime structures. These local-to-global procedures procedures will therefore lead to a wide range of applications sketched in the later sections, such as the *emergence of higher dimensional spacetime* structures through highly complex dynamics in organismic development, adaptation, evolution, consciousness and society interactions.

Classical physics, including GR involves a concept of both *continuous* and *homogeneous* space and time with strict causal (mechanistic) evolution of all physical processes ("*God does not play dice*", cf. Albert Einstein). Furthermore, up to the introduction of *quanta–discrete* portions, or packets–of energy by Ernst Planck (which was further elaborated by Einstein, Heisenberg, Dirac, Feynman, Weyl and other eminent physicists of the last century), energy was also considered to be a continuous function, though not homogeneously distributed in space and time. Einstein's Relativity theories joined together space and time into one 'new' entity–the concept of *spacetime.* In the improved form of GR, inhomogeneities caused by the presence of matter are also allowed to occur in spacetime. Causality, however, remained *strict*, but also more complicated than in the Newtonian theories as discontinuities appear in spacetime in the form of singularities, or 'black holes. The standard GR theory, the Maxwellian Theory of Electromagnetism and Newtonian mechanics can all be considered *Abelian*, even though GR not only allows, but indeed, requires spacetime inhomogeneities to occur in the presence of gravitational fields, unlike Newtonian mechanics *where space is both absolute and homogeneous. Recent efforts to develop non-Abelian* GR theories–especially with an intent to develop Quantum Gravity theories– seem to have considered both possibilities of locally homogeneous or inhomogeneous, but still globally continuous spacetimes. The successes of non-Abelian gauge theories have become well known in physics since 1999, but they still await the experimental discovery of their predicted Higgs boson particles [267].

Although Einstein's Relativity theories incorporate the concept of *quantum of energy*, or photon, into their basic structures, they also deny such discreteness to spacetime even though the discreteness of energy is obviously accepted within Relativity theories. The GR concept of spacetime being modified, or *distorted/'bent'*, by matter goes further back to Riemann, but it was Einstein's GR theory that introduced the idea of representing gravitation as the result of *spacetime distortion by matter.* Implicitly, such spacetime distortions remained continuous even though the gravitational field energy –as all energy– was allowed to vary in *discrete*, albeit very tiny portions–the gravitational quanta. So far, however, the detection of gravitons –the quanta of gravity–related to the spacetime distortions by matter–has been unsuccessful. Mathematically elegant/precise and physically 'validated' through several crucial experiments and astrophysical observations, Einstein's GR is obviously not reconcilable with Quantum theories (QTs). GR was designed as the *large–scale* theory of the Universe, whereas Quantum theories–at least in the beginning–were designed to address the problems of *microphysical* measurements at very tiny scales of space and time involving extremely small quanta of energy. We see therefore the QTs vs. GR as a local-to-global problem that has not been yet resolved in the form of an universally valid Quantum Gravity. Promising, partial solutions are suggested in three recent papers [36],[38], [70]. Quantum theories (QTs) were developed that are just as elegant mathematically as GR, and they were also physically 'validated' through numerous, extremely sensitive and carefully designed experiments.

However, to date quantum theories have not yet been extended, or generalized, to a form capable of recovering the results of Einstein's GR as a quantum field theory over a GR-spacetime altered by gravity. Furthermore, quantum symmetries occur not only on microphysical scales, but also macroscopically in certain, 'special' cases, such as liquid ^{3}He close to absolute zero and superconductors where *extended coherence* is possible for the superfluid, long-range coupled Cooper electron-pairs. However, explaining such interesting physical phenomena also requires the consideration of *symmetry breaking* resulting from the Goldstone Boson Theorem as it was shown in [267]. Occasionally, symmetry breaking is also invoked in the recent science literature as a 'possible mechanism for human consciousness' which also seems to be related to, or associated with some form of 'global coherence'–over most of the brain; however, the existence of such a '*quantum* coherence in the brain'–at least at physiological temperatures–as it would be precisely required/defined by QTs, is a most unlikely event. On the other hand, a *quantum symmetry breaking* in a neural network considered metaphorically as a Hopfield ('spin-glass') network might be conceivable close to physiological temperatures, except for the lack of evidence of the existence of any requisite (electron) spin lattice structure that is indeed an absolute requirement in such a spin-glass metaphor.

Now comes the real, and very interesting part of the story: neuronal networks do form functional patterns and structures that possess partially 'broken', or more general symmetries than those described by quantum groups. Such *extended symmetries* can be mathematically determined, or specified, by certain *groupoids*–that were previously called *'neuro-groupoids' [33].* Even more generally, genetic networks also exhibit extended symmetries that are present in biological species which are represented by a *biogroupoid* structure, as previously defined and discussed by Baianu, Brown, Georgescu and Glazebrook in [32]-[33]. Such biogroupoid structures [33] can be experimentally validated, for example, at least partially through Functional Genomics observations and computer, bioinformatics processing [30]. We shall discuss further several such interesting groupoid structures in the following sections, and also how they have already been utilized in so-called 'local-to-global procedures' in order to construct 'global' solutions; such global solutions in quite complex (holonomy) cases can still be *unique* up to an isomorphism (*the Globalisation Theorem*, as it was discussed in [69], and references cited therein. Last-but-not-least, *holonomy* may provide a global solution, or 'explanation for memory storage by 'neurogroupoids'. Uniqueness holonomy theorems might possibly explain the existence of unique, persistent and resilient memories.

Towards Biological Postulates and Principles

Whereas the hierarchical theory of levels provides a powerful, systems approach through categorical ontology, the foundation of science involves *universal* models and theories pertaining to different levels of reality. It would seem natural to expect that theories aimed at different ontological levels of reality should have different principles. We are advocating the need for developing precise, but nevertheless 'flexible', concepts and novel mathematical representations suitable for understanding the emergence of the higher complexity levels of reality. Such theories are based on axioms, principles, postulates and laws operating on distinct levels of reality with a specific degree of complexity. Because of such distinctions, inter-level principles or laws are rare and over-simplified principles abound. Alternative approaches may be, however, possible based upon an improved ontological theory of levels. Interestingly, the founder of Relational Biology, Nicolas Rashevsky proposed in 1969 that physical laws and principles can be expressed in terms of *mathematical functions*, or mappings, and are thus being predominantly expressed in a *numerical* form, whereas the laws and principles of biological organisms and societies need take a more general form in terms of quite general, or abstract–mathematical and logical relations which cannot always be expressed numerically; the latter are often qualitative, whereas the former are predominantly quantitative [224].

Rashevsky focused his Relational Biology/Society Organization papers on a search for more general relations in Biology and Sociology that are also compatible with the former. Furthermore, Rashevsky proposed two biological principles that add to Darwin's natural selection and the 'survival of the fittest principle', *the emergent relational structure that are defining the adaptive organism*:

1. The Principle of Optimal Design[233],

and

2. The Principle of Relational Invariance (initially phrased by Rashevsky as *"Biological Epimorphism"*)[12]-[13],[15],[222].

In essence, the 'Principle of Optimal Design' [233] defines the organization and structure of the 'fittest' organism which survives in the natural selection process of competition between species, in terms of an extremal criterion, similar to that of Maupertuis; the optimally 'designed' organism is that which acquires maximum functionality essential to survival of the successful species at the lowest 'cost' possible [11]-[13]. The 'design' in this case is commonly taken in the sense of the result of a long evolutionary process that occurred under various environmental and propagation constraints or selection 'pressures', such as that caused by sexual reproduction in Darwin's model of the origin of species during biological evolution. The 'costs' are here defined in the context of the environmental niche in terms of material, energy, genetic and organismic processes required to produce/entail the pre-requisite biological function(s) and their supporting anatomical structure(s) needed for competitive survival in the selected niche. Further details were presented by Robert Rosen in his short, but significant, book on optimality principles in theoretical biology [233], published in 1967.

The 'Principle of Biological Epimorphism', on the other hand, states that the highly specialized biological functions of higher organisms can be mapped (through an epimorphism) onto those of the simpler organisms, and ultimately onto those of a (hypothetical) primordial organism (which is assumed to be unique up to an isomorphism or *selection-equivalence*). The latter proposition, as formulated by Rashevsky, is more akin to a postulate than a principle. However, it was then generalised and re-stated as the Postulate of Relational Invariance [12]. Somewhat similarly, a dual principle and the colimit construction were invoked for the ontogenetic development of organisms [11], and more recently other quite similar colimit constructions were considered in relation to 'Memory Evolutive Systems', or phylogeny [103]-[104].

An axiomatic system (ETAS) leading to higher dimensional algebras of organisms in supercategories has also been formulated [18] which specifies both the logical and the mathematical ($\pi-$) structures required for complete self-reproduction and self-reference, self-awareness, etc. of living organisms. To date, there is no higher dimensional algebra (HDA) axiomatics other than the ETAS proposed for complete self-reproduction in super-complex systems, or for self-reference in ultra-complex ones. On the other hand, the preceding, simpler ETAC axiomatics introduced by Lawvere, was proposed for the foundation of 'all' mathematics, including categories [166]-[167], but this seems to have occurred before the actual emergence of HDA.

6. Towards a Formal Theory of Levels in Ontology

This subsection will introduce in a concise manner fundamental concepts of the ontological theory of levels. Further details were reported by Poli in [206]-[211], and by Baianu and Poli in this volume [40].

Fundamentals of Poli's Theory of Levels

The ontological theory of levels by Poli [206]-[211] considers a hierarchy of *items* structured on different levels of reality, or existence, with the higher levels *emerging* from the lower, but usually *not* reducible to the latter, as claimed by widespread reductionism. This approach modifies and expands considerably earlier work by Hartmann [137] both in its vision and the range of possibilities. Thus, Poli in [206]-[211] considers four realms or *levels* of reality: Material-inanimate/Physico-chemical, Material-living/Biological, Psychological and Social. Poli in [211] has stressed a need for understanding *causal and spatiotemporal* phenomena formulated within a *descriptive categorical context* for theoretical levels of reality. There is the need in this context to develop a *synthetic* methodology in order to compensate for the critical ontic data analysis, although one notes (cf. Rosen in 1987 [232]) that analysis and synthesis are not the exact inverse of each other. At the same time, we address in categorical form the *internal dynamics*, the *temporal rhythm, or cycles*, and the subsequent unfolding of reality. The genera of corresponding concepts such as 'processes', 'groups', 'essence', 'stereotypes', and so on, can be simply referred to as *'items'* which allow for the existence of many forms of causal connection [210]-[211]. The implicit meaning is that the *irreducible multiplicity* of such connections converges, or it is ontologically integrated within a *unified synthesis*.

The Object-based Approach vs Process-based (Dynamic) Ontology

In classifications, such as those developed over time in Biology for organisms, or in Chemistry for chemical elements, the *objects* are the basic items being classified even if the 'ultimate' goal may be, for example, either evolutionary or mechanistic studies. An ontology based strictly on object classification may have little to offer from the point of view of its cognitive content. It is interesting that D'Arcy W. Thompson arrived in 1941 at an ontologic "*principle of discontinuity*" which "is inherent in all our classifications, whether mathematical, physical or biological... In short, nature proceeds *from one type to another* among organic as well as inorganic forms... and to seek for stepping stones across the gaps between is to seek in vain, for ever." (p.1094 of Thompson in [259], re-printed edition). Whereas the existence of different ontological levels of reality is well-established, one cannot also discard the study of emergence and co-emergence processes as a path to improving our understanding of the relationships among the ontological levels, and also as an important means of ontological classification. Furthermore, the emergence of ontological meta-levels cannot be conceived in the absence of the simpler levels, much the same way as the chemical properties of elements and molecules cannot be properly understood without those of their constituent electrons.

It is often thought that the *object-oriented* approach can be readily converted into a process-based one. It would seem, however, that the answer to this question depends critically on the ontological level selected. For example, at the quantum level, *object and process become inter-mingled.* Either comparing or moving between levels– for example through emergent processes– requires ultimately a *process-based* approach, especially in Categorical Ontology where relations and inter-process connections are essential to developing any valid theory. Ontologically, the quantum level is a fundamentally important starting point which needs to be taken into account by any theory of levels that aims at completeness. Such completeness may not be attainable, however, simply because an ‘extension’ of Gödel’s theorem may hold here also. The fundamental quantum level is generally accepted to be dynamically, or intrinsically *non-commutative*, in the sense of the *non-commutative quantum logic* and also in the sense of *non-commuting quantum operators* for the essential quantum observables such as position and momentum. Therefore, any comprehensive theory of levels, in the sense of incorporating the quantum level, is thus –*mutatis mutandis*– *non-Abelian.* A paradigm shift towards a *non-Abelian Categorical Ontology* has already begun [33]-[34],[37]-[38],[40],[69].

From Component Objects and Molecular/Anatomical Structure to Organismic Functions and Relations: A Process–Based Approach to Ontology

Wiener in 1950 made the important remark that implementation of *complex functionality* in a (complicated, but not necessarily complex–in the sense defined above) machine requires also the design and construction of a correspondingly *complex structure*, or structures [269]. A similar argument holds *mutatis mutandis*, or by induction, for *variable* machines, variable automata and variable dynamic systems [12]-[23]; therefore, if one represents organisms as variable dynamic systems, one *a fortiori* requires a *super-complex structure* to enable or entail *super-complex dynamics*, and indeed this is the case for organisms with their extremely intricate structures at both the molecular and *supra-molecular* levels. This seems to be a key point which appears to have been missed in the early-stages of Robert Rosen’s theory of simple (M, R)-systems, prior to 1970, that were deliberately designed to have “no structure” as it was thought they would thus attain the highest degree of generality or abstraction, but were then shown by Warner to be equivalent to a special type of sequential machine or classical automaton [17],[264].

The essential properties that define the super– and ultra– complex systems derive from the *interactions, relations and dynamic transformations* that are ubiquitous at such levels of reality– which need to be distinguished from the levels of organization internal to any biological organism or biosystem. Therefore, a complete approach to Ontology should obviously include *relations and interconnections* between items, with the emphasis on *dynamic processes, complexity* and *functionality* of systems. This leads one to consider general relations, such as *morphisms* on different levels, and thus to the *categorical viewpoint* of Ontology. The *process-based approach* to an Universal Ontology is therefore essential to an understanding of the Ontology of Reality Levels, hierarchies, complexity, anticipatory systems, Life, Consciousness and the Universe(s). On the other hand, the opposite approach, based on objects, is perhaps useful only at the initial cognitive stages in experimental science, such as the simpler classification systems used for efficiently organizing data and providing a simple data structure. We note here also the distinct meaning of ‘object’ in psychology, which is much different from the one considered in this subsection; for example, an external process can be ‘reflected’ in one’s mind as an ‘object of study’. This duality, or complementarity between ‘object’ and ‘subject’, ‘objective’ and ‘subjective’ seems to be widely adopted in philosophy, beginning with Descartes and continuing with Kant, Heidegger, and so on. A somewhat similar, but not precisely analogous distinction is fundamental in standard Quantum Theory– the distinction between the observed/measured system (which is the quantum, ‘subject’ of the measurement), and the measuring instrument (which is a classical ‘object’ that carries out the measurement).

Physicochemical Structure–Function Relationships

It is generally accepted at present that structure-functionality relationships are key to the understanding of super-complex systems such as living cells and organisms. Integrating structure–function relationships into a Categorical Ontology approach is undoubtedly a viable alternative to any level reduction, and philosophical/epistemologic reductionism in general. Such an approach is also essential to the science of complex/super-complex systems; it is also considerably more difficult than either physicalist reductionism, entirely *abstract relationalism* or ‘rhetorical mathematics’. Moreover, because there are many alternative ways in which the physico-chemical structures can be combined within an organizational map or relational complex system, there is a *multiplicity of ‘solutions’* or mathematical models that needs be investigated, and the latter are not computable with a digital computer in the case of complex/super-complex systems such as organisms [23],[232]. The problem is further compounded by the presence of *structural disorder* (in the physical structure sense) which leads to a very high *multiplicity* of dynamical-physicochemical structures (or ‘configurations’) of a biopolymer– such as a protein, enzyme, or nucleic acid, of a biomembrane, as well as of a living cell, that correspond to a single function or a small number of physiological functions [20]; this complicates the assignment of a ‘fuzzy’ physico-chemical structure to a well-defined biological function unless extensive experimental data are available, as for example, those derived through computation from 2D-NMR spectroscopy data (as for example by Wütrich, in 1996 [271]), or neutron/X-ray scattering and related multi-nuclear NMR spectroscopy/relaxation data [20] Detailed considerations of the ubiquitous, or universal, partial disorder effects on the structure-functionality relationships were reported for the first time by Baianu in 1980 [20]. Specific aspects were also recently discussed by Wütrich in 1996 on the basis of 2D-NMR analysis of ‘small’ protein configurations in solution [271].

As befitting the situation, there are devised *universal* categories of reality in its entirety, and also subcategories which apply to the respective subdomains of reality. We harmonize this theme by considering categorical models of complex systems in terms of an evolutionary dynamic viewpoint using the mathematical methods of Category Theory which afford describing the characteristics, classification and emergence of levels, besides the links with other theories that are, *a priori*, essential requirements of any ontological theory. We also underscore a significant component of this essay that relates the ontology to geometry/topology; specifically, if a level is defined via ‘iterates of local procedures' (cf 'items in iteration' cf. Brown and İçen in [71]), that will further expanded upon in the last sections; then we will have a handle on describing its intrinsic governing dynamics (with feedback). As we shall see in the next subsection, categorical techniques– which form an integral part of our ontological considerations– provide a means of describing a hierarchy of levels in both a linear and interwoven, or *entangled*, fashion, thus leading to the necessary bill of fare: emergence, higher complexity and open, non-equilibrium/irreversible systems. We must emphasize that the categorical methodology selected here is *intrinsically 'higher dimensional'*, and can thus account for meta–levels, such as 'processes between processes...' within, or between, the levels–and sub-levels– in question. Whereas a strictly Boolean classification of levels allows only for the occurrence of *discrete* ontological levels, and also does not readily accommodate either *contingent* or *stochastic sub-levels*, the LM-logic algebra is readily extended to *continuous*, *contingent* or even *fuzzy* sub-levels, or levels of reality [11],[23],[32]-[34],[39]-[40],[120],[140]. Clearly, a Non-Abelian Ontology of Levels would require the inclusion of either Q- or LM- logics algebraic categories (discussed in the following section) because it begins at the fundamental quantum level –where Q-logic reigns– and 'rises' to the emergent ultra-complex level(s) with 'all' of its possible sub-levels represented by certain LM-logics. (Further considerations on the meta–level question are presented by Baianu and Poli in this volume [40]). On each level of the ontological hierarchy there is a significant amount of connectivity through inter-dependence, interactions or general relations often giving rise to complex patterns that are not readily analyzed by partitioning or through stochastic methods as they are neither simple, nor are they random connections. This ontological situation gives rise to a wide variety of networks, graphs, and/or mathematical categories, all with different connectivity rules, different types of activities, and also a hierarchy of super-networks of networks of subnetworks. Then, the important question arises *what types of basic symmetry or patterns* such super-networks of items can have, and how do the effects of their sub-networks percolate through the various levels. From the categorical viewpoint, these are of two basic types: they are either *commutative* or *non-commutative*, where, at least at the quantum level, the latter takes precedence over the former, as we shall further discuss and explain in the following sections.